Exploiting temporal noises and device fluctuations in enhancing fidelity of pulse-density modulator consisting of single-electron neural circuits

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Abstract. This paper discusses the implications of noises in a pulse-density modulation single-electron circuit based on Vestibulo-ocular Reflex model. The proposed circuit consists of an ensemble of single-electron integrate-and-fire neurons that encode the input voltage into pulses whose temporal density is proportional to the amplitude of the input. We confirmed that static noises (heterogeneity in circuit parameters) and dynamic noises (random firing) introduced into the network indeed played an important role in improving the fidelity with which the neurons could encode signals with input frequencies higher than the intrinsic response frequencies of single neurons or a network of neurons without noises. Through Monte-Carlo based computer simulations, we demonstrated that noises could enhance the fidelity with which the network could correctly encode signals with high input frequencies: a noisy network could operate over a wider input range that a single neuron or a network of homogeneous neurons.

Key words: neuromorphic LSIs; neural networks; single-electron circuits; pulse-density modulation

1 Introduction

As physical features of electronic devices approach the deep sub-micron (nano) scales, variance in physical parameters of fabricated devices (static noises) and sensitivity to external noises (dynamic noises) become more pronounced, posing a challenging task to the circuit designer. Most of the research toward solving these problems has been focused on reducing the impact of static noises through improved fabrication techniques, improving shielding technologies to protect devices from radiation and extrenal noises, or architectural level approaches where additional circuitry is introduced into the system to increase the signal to noise ratio. The above-mentioned approaches might not provide the once-and-future

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solution, especially for the constantly shrinking device sizes. A novel approach to solving this problem would be to effectively use both static and dynamic noises to improve circuit performance.

If we look at how signal processing is carried out in neuronal systems, we find that individual neurons have high heterogeneity in intrinsic response properties; they have diverse variances in firing rates, and some of the neurons are even defective. However, in spite of these setbacks neuronal systems accurately encode signals as they are relayed from sensory organs to the central nervous system, or to other organs. A number of reports suggest that neurons in fact employ heterogeneity to effectively encode signals ([1] - [3]). Hospedales et al. ([1]) demonstrated that neurons in the medial vestibular nucleus (MVN) can encode high frequency signals with a high temporal precision as a result of their heterogeneity. This paper introduces a neuromorphic circuit that effectively utilizes both static and dynamic noises to improve the temporal fidelity of signal transmission in a pulse density modulation circuit based on Vestibulo-ocular Reflex (VOR) model.

The paper is organised as follows. A short review of pulse-density modulation in integrate-and-fire neurons is presented. This is folowed by the noisy network model and its circuit implementation with single-electron devices. Thirdly, the model and circuit configuration are explained. Finally the validity of the model is verified with Monte-Carlo based simulations.

2 Pulse-density modulation in integrate-and-fire neurons

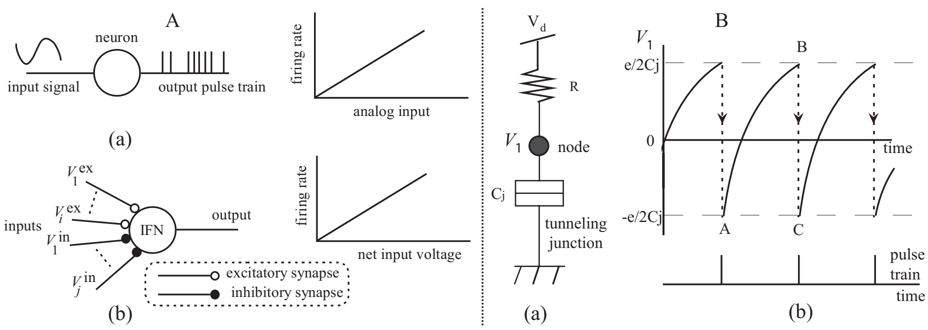

Fig. 1. A:(a) Pulse density modulation in integrate-and-fire neurons: analog input is converted into a pulse train (b) Fundamental structure and operation of integrate-and-fire neurons (IFNs). The IFN receives input voltages through excitatory and inhibitory synapses, and produces a pulse train whose pulse density (firing rate) is proportional to the net input voltage. B: Single-electron tunneling (SET) oscillator: (a) circuit structure and (b) waveform showing oscillation.

An integrate-and-fire neuron (IFN) aggregates inputs from other neurons connected through synapses. The aggregated charge raises the membrane potential until it reaches a threshold, where the neuron fires generating a spike. This spike corresponds to a binary output " 1 ". After the firing event, the membrane potential is reset to a low value, and it increases again as the neuron accepts inputs from neighboring neurons (or input signals) to repeat the same cycle; producing a stream of " one " and " zero " pulse trains. The spike interval (density of spikes per unit time) is proportional to the analog input voltage i.e. the level of analog input is coded into pulse density. Thus a neuron can be considered as a 1-bit A-D converter operating in the temporal domain. Fig. 1A:(a) shows a schematic representation of analog-to-digital conversion in IFNs. The output pulse density is proportional to the amplitude of the input signal. Fig. 1A:(b) shows the fundamental operation of an IFN. The open circles ($\circ$) and shaded circles ($\bullet$) represent excitatory and inhibitory synapses, respectively. The IFN receives input signals (voltages) through the excitatory synapses (to raise its membrane potential) and inhibitory synapses (which decrease the membrane potential) from adjacent neurons, to produce a spike if the postsynaptic potential ($\sum V_i^{\rm ex} - \sum V_j^{\rm in}$) exceeds the threshold voltage. After the IFN fires, its membrane voltage is reset to a low value, and the integration action resumes.

3 Single-electron Integrate and Fire Neuron

The operation of an integrate-and-fire neuron (IFN) is modelled with a single-electron oscillator [4] - [5]. A single-electron oscillator (Fig. 1B:(a)) consists of a tunneling junction (capacitance = $C_{\rm j}$) and a high resistance R connected in series at the nanodot ($\bullet$) and biased with a positive or a negative voltage $V_{\rm d}$. It produces self-induced relaxation oscillations if the input voltage is higher than the tunneling threshold ($V_{\rm d} > e/(2C_{\rm j})$) where e is the elementary charge and k_B is the Boltzmann constant. The naodot voltage V_1 increases as the capacitance $C_{\rm j}$ is charged through the series resistance (curve AB), until it reaches the tunneling threshold $e/(2C_{\rm j})$, at which an electron tunnels from the ground to the nanodot across the tunneling junction, resetting the nanodot voltage to $-e/(2C_{\rm j})$. This abrupt change in nanodot potential (from B to C) can be referred to as a firing event. The nanodot is recharged to repeat the same cycles. Therefore, a single-electron oscillator could be viewed as an integrate and fire neuron, which aggregates inputs (or inputs from from neighboring neurons) producing a pulse when its nanodot voltage reaches the threshold voltage (Fig. 1B:(b)). By feeding a sinusoidal input to a single-electron oscillator, one can adjust the probability of electron tunneling in the circuit: the tunneling rate increases as the input voltage rises above the threshold and gradually decreases to zero as the input approaches and falls below the threshold value. In other words, a single-electron oscillator converts an analog input into digital pulses. A single-electron oscillator can thus be viewed as a pulse-density modulator (PDM), that produces a spike train (or produces zero) if the input signal exceeds (or falls below) the threshold value.

4 Model and circuit structure

The single-electron integrate-and-fire neuron explained in the preceding section is used to study the implications of noises enhancing fidelity of signal transmission in a neuronal single-electron circuit. The circuit is based on a model of the vestibulo-ocular reflex (VOR) proposed by Hospedales et al. ([1]). In their work, they reported that noises and heterogeneity in the intrinsic response properties of neurons account for the high-fidelity in VOR functionality.

Fig. 2(a) shows the part of the model, which converts head movements into neural spikes in the VOR, consisting of n neurons. The structural heterogeneity in the synaptic couplings (membrane time constants) of individual neurons is represented by ξ_i. We refer to this heterogeneity as static noises. The neurons receive a common analog input and produce spikes whose temporal density corresponds to the amplitude of the input signal. The output terminal receives pulses from all the neurons in the network to produce a spike train. The noises introduced into the network lead to random and independent firing events in the neurons, reducing the probability of synchrony in the network. In addition, the variations in parameters increases the randomness with which the network neurons fire, increasing the probability of a ready-to-fire neuron at any given time, which consequently enhances the precision with which the neurons in the network can encode signals with input frequencies higher those of individual neurons.

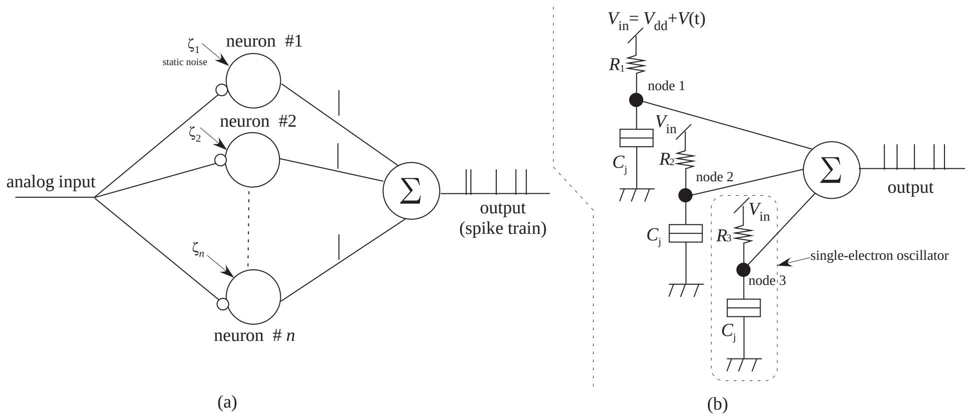

Fig. 2. (a) Neural network model of signal encoding in the VOR consisting of n neurons, (b) Implementation with single-electron oscillators.

The network is implemented with single-electron IFNs (oscillators) as shown in Fig. 2(b). The heterogeneity in the model was introduced in the circuit as variations in the series resistance R. Note that R is a critical parameter in setting the intrinsic response frequency of each neuron. Therefore, by tuning the values of R, we could simulate the heterogeneity of membrane time constants of actual neurons.

5 Simulation results

In the simulations, the single-electron neurons were fed with a common input voltage $V_{\rm in} = V_{\rm dd} + V({\rm t})$, where $V_{\rm dd}$ (bias voltage) was set to 7.8 mV to achieve a monostable operation in the absence of input signals, $V({\rm t})$ is a pulsed input voltage with an amplitude of 0.8 mV. The capacitance of the tunneling junctions $C_{\rm j}$ was set to 10 aF. The simulation time was set to 800 ns, while the operation temperature (T) was set to 0.5 K for simulation results shown in Figs. 3, 4 and 5(b) and (c).

Fig. 3 shows the transient response of a unit single-electron neuron. Fig. 3(a) and (c) show the input signals with a frequency of 600 MHz and 250 MHz, respectively. Fig. 3(b) shows the neuron response to input "(a)", while "(d)" shows the neuron respone to input "(c)". The series resistance was set to 100 MΩ. Fig. 3(d) shows successful encoding of the input signal (the neuron fires once for each pulse in the input signal[1]) whose frequency is within the intrinsic firing rate of a single neuron. In Fig. 3(b), the neuron could only trasmit some of the input pulses, leading to a lower firing rate as compared to the input rate. In other words, the neuron in (b) could only transmit some of the input pulses toward the output. This degrades the fidelity of signal transmision along the neural network.

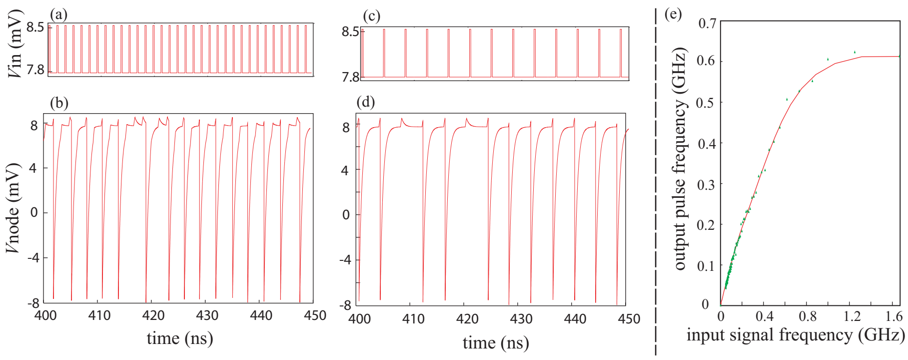

Fig. 3. Transient response of a single neuron. (a) and (c) show input signals with input frequencies of 600 MHz and 250 MHz, respectively. (b) and (d) show the output characteristics of neurons fed with input signals of 600 MHz and 250 MHz, respectively. (e) Output firing rate of a single neuron plotted against the input pulse frequency.

Fig. 3(e) shows the response of a single neuron over a wide range of input frequencies. The horizontal axis shows the input frequency, while the vertical

[1] Tunneling (firing) in single-electron devices involves a probabilistic time lag or waiting time between when the node voltage exceeds the threshold voltage and when an electron can actually tunnel from the ground to the node, sending a spike toward the output terminal. Due to the effect of the time lag, a neuron might fail to fire even after achieving the tunneling conditions as seen in Fig. 3(d). As a result, the average firing rate would be somewhat lower than the input pulse rate

axis shows the average firing rate of the neuron. The neuron response was linear for input signals with a frequency of upto 0.5 GHz. Beyond this range, the output was highly distorted. This shows that a single neuron can successfully encode (respond to) signals with a maximum input frequency of 500 MHz.

The response of a population of neurons to various input frequencies was investigated with two sets of neuron ensembles: homogeneous and heterogeneous networks. In the homogeneous ensemble, the series resistances R_1, R_2, and R_3 were set to the same value, whereas in the second set, heterogeneity (static noises) was introduced by varying the values of series resistances in the three neurons. The results are shown in Figs. 4 and Fig. 5.

Fig. 4(a) shows the input signal with a frequency of 600 MHz. Figs. 4(b1) and (c-1) show the response of the homogeneous network, where the series resistances R_1, R_2 and R_3 were set to 100 MΩ. Fig. 4(b1) shows the firing events of individual neurons in the network. Fig. 4(c1) shows the summed spike output (spike train) at the output terminal. We could confirm that the neurons in the homogeneous network tend to synchronize, firing at almost the same timing.

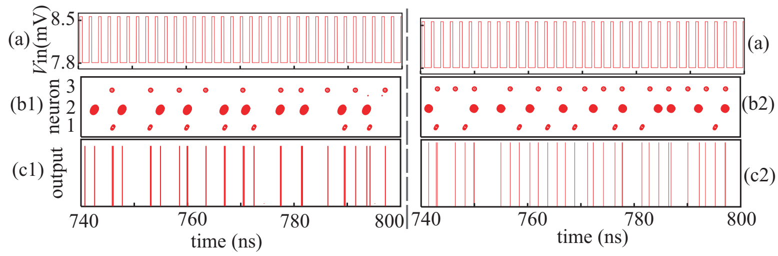

Fig. 4. Transient responses of both homogeneous and heterogeneous networks. (a) shows the input signal. (b1) shows the firing events of each neuron, while (c1) whows the summed pulse output for the three neurons in the homogeneous network. (b2) shows the firing events, and (c2) shows the summed pulse output of the heterogeneous network.

Figs. 4 (b2) and (c2) show the response of neurons in the heterogeneous network, where the series resistances were set to 110 MΩ for neuron 1, 100 MΩ for neuron 2 and 90 MΩ for neuron 3. The firing events in the heterogeneous network are more or less random as shown in Fig. 4(b2). The probability of having a neuron with a potential near the threshold value, at any given moment, is higher than in the case of a homogeneous network. Thus the network can respond to any incoming pulses at a higer probability. This results in an improved encoding of the input as illustrated by the spike train shown in Fig. 4(c2). In other words, since the neurons fired irregularly, they could transmit the input pulses with a higher temporal precision as opposed to the homogeneous network. This is elaborated in more detail in Fig. 5 (curves (b) and (c)), where the transmission of signal over a wide range of frequencies is demonstrated. The horizontal axis represents the frequency of input signals, while the vertical axis shows the average firing rate (output frequency) for both neuron sets. In the case of the homogenous network, since the neurons tend to synchronize with time, their encoding frequency is the same as that of individual neurons. Contrary, neurons in the heterogeneous network could correctly encode signals with input frequencies upto 1 GHz, twice that of the homogeneous network. This demonstrates that heterogeneity in the circuit parameters (presence of static noises) plays an important role in improving the fidelity with which neurons can encode signals with input frequencies far beyond the encoding capacity of individual neurons.

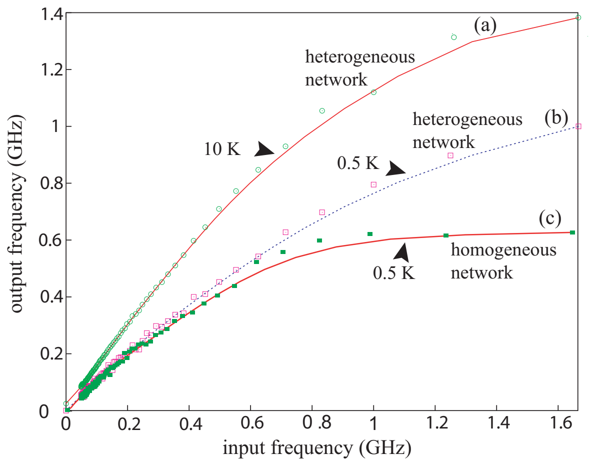

Fig. 5. Output firing rate of an ensemble of neurons plotted against the input pulse frequency. (a) and (b) show response characteristics of a heterogeneous network simulated at a temperature of 10 K and 0.5 K, respectively. (c) shows response characteristics of a homogeneous network simulated at 0.5 K.

6 Effect of dynamic noises

Hospedales et al. ([1]) investigated the importance of random noises in improving the fidelity of signal transmission in the VOR response. They concluded that besides neuronal heterogeneity, externally induced noises also play an important role in improving the network performance. These external noises could be as a result of spontaneous increases or decreases of membrane potential due to firing events in other neurons in the network. These changes are random and are often referred to as dynamic noises. In our circuit, we studied the effect of dynamic noises by considering thermally induced tunneling events in the network. Curves(a) and (b) in Fig. 5 show the response characteristics of a network simulated at 10K, and 0.5K, respectively. As the temperature increases, thermally induced tunneling events in single-electron neurons increase, resulting in an increase in the average firing rate in the network. This is illustrated by the increased firing rate at a temperature of 10 K. Although this work suggests that dynamic noises don't play a critical role in increasing the maximum response frequency of the network, they however, increase the fidelity with which the network can sample input signals within the maximum input signal frequency range determined by heterogeneity in the network elements. This is evident at higher input frequencies, where the ratio of the output pulse rate to the input pulse rate starts to roll-off rapidly. The roll off is compensated for by the dynamic noises, which reduces the effect of waiting time in electron tunneling.

7 Conclusion

In this study, we proposed and investigated the implication of heterogeneity in transmission of high frequency signals in a neural network. Through Monte-Carlo based computer simulations, we confirmed that heterogeneity in device parameters indeed improved the temporal precision with which the network could transmit signals with high input frequencies within the network. A heterogeneous network could correctly encode signals of upto 1 GHz, as compared to 500 MHz in single neurons (or a network of homogenous neurons). Another important factor to consider in improving the fidelity of this circuit would be the effect of external and internal (dynamic) noises. In single-electronic devices, such noises include thermally induced random firing events or the effect of environmental noises. As we have shown, as the temperature increases, the dynamic noises also increase compensate for the roll-off in response of the network, especially at high frequencies. Although a comprehensive investigation on the implications of dynamic noises to signal transmission is required, the preliminary results presented in this paper show that in addition to heterogeneity in neuron properties, externally introduced noises could assist in further improving the fidelity of signal encoding in single-electron circuits. We should however, note that at higher temperatures, beyond the results presented here, random tunneling as a result of dynamic noises would increase rapidly leading to degradation of signal transmission. Therefore, the value of dynamic noises to be introduced to the network to achieve the best performance needs to be optimized.

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