

Excitable Reaction-Diffusion Media with Memristors

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We investigated spatiotemporal behaviors of excitable reaction-diffusion (RD) system where excitable Oregonators are coupled by *memristors*.

A 1-D reaction-diffusion system can be described by

$$\begin{aligned}\frac{\partial u(x)}{\partial t} &= g_u \nabla^2 u(x) + f_u[u(x), v(x)], \\ \frac{\partial v(x)}{\partial t} &= g_v \nabla^2 v(x) + f_v[u(x), v(x)],\end{aligned}$$

where $u(x)$ and $v(x)$ represent the concentration at spatial position x , $g_{u,v}$ the diffusion coefficients, and $f_{u,v}(\cdot)$ the reaction model. We here employ Oregonators for the reaction model, and consider excitable properties ($g_v = 0$ only). Although g_u is constant in general RD models, we are interested in a system where g_u is locally modified by the potential gradient of $u(x)$. Hence we spatially discretize Eq. (1), and introduce the following dynamics:

$$\dot{u}_i = \frac{g_u(w_i^l)(u_{i-1} - u_i) + g_u(w_i^r)(u_{i+1} - u_i)}{\Delta x^2} + f_u(\cdot),$$

where i is the spatial index, Δx the discrete step in space, and $g(\cdot)$ the coupling function defined by

$$g_u(w_i^{l,r}) = g_{\min} + (g_{\max} - g_{\min}) \cdot \frac{1}{1 + e^{-\beta w_i^{l,r}}},$$

where β represents the gain, $g_{\min, \max}$ the minimum and maximum coupling strengths, and $w_i^{l,r}$ the variables for determining the coupling strength of the i -th Oregonator (l : leftward, r : rightward). We here assume the following memristive dynamics for $w_i^{l,r}$:

$$\tau \frac{dw_i^{l,r}}{dt} = g_u(w_i^{l,r}) \cdot (u_{i-1,i+1} - u_i).$$

The model above corresponds to an electrical RD system consisting of Oregonators whose diffusive resistors are replaced with memristors (Fig. 1 right).

Figure 2(a) shows simulation results of the model with 100 Oregonators without memristive effects, exhibiting excitable wave propagation on the medium. Both the boundary were simultaneously stimulated, and the waves were collided at the center position (then they disappeared). When we introduced memristive effects here, since coupling strength $g_u(w_i^{l,r})$ is modified by direction of the wave propagation, the result would

be different from Fig. 2(a). Figure 2(b) shows simulation results of the model with memristive effects, where velocities of each excitable wave was different due to the direction of the wave propagation, which resulted in wave collision at non-center position. Excitable waves moving rightward (in the figure) increased $w_i^{l,r}$ of memristors under the wave, whereas rightward waves decreased $w_i^{l,r}$ under the wave, which resulted from polarity of memristors shown in Fig. 1 (left). We will further investigate spatiotemporal properties of the model having i) random replacement (polarity) of memristors and ii) 2-D version of the model.

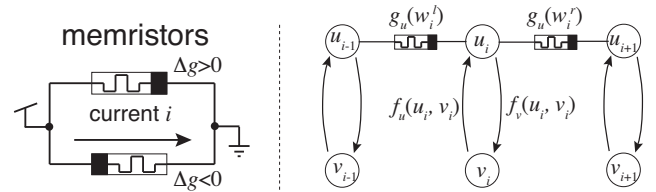
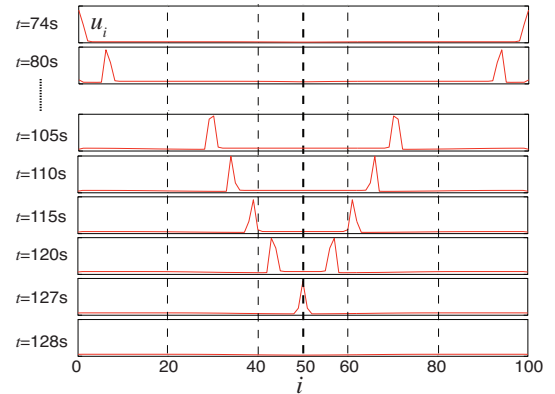
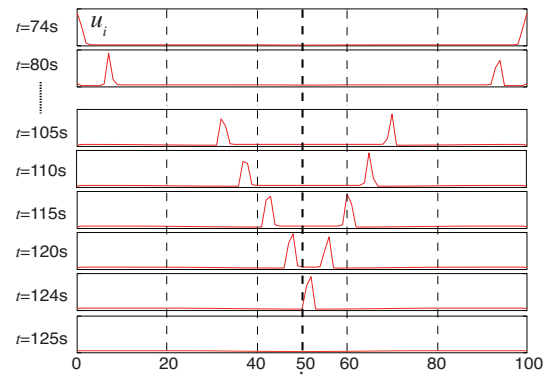


Fig. 1. Proposed memristor STDP circuit.



(a) Excitable wave propagation on non-memristive media



(b) Excitable wave propagation on memristive media

Fig. 2. Simulation results (time vs space).

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