CIRCUIT IMPLEMENTATION OF HISTORIC ANALOG CELLULAR AUTOMATA BASED ON WOLFRAM'S RULES 90 AND 150

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Standard cellular automata (CA) employes transition rules where a cell's state is updated in terms of the previous state (1 step before) of the cell's neighbors. This standard framework has recently been expanded by introducing historic memory capabilities into CAs, i.e., a cell's state is updated by not only the neighbor's previous state but also the past states (2, 3, ..., M steps before), which resulted in generating statistically good random numbers [1], spatialized prisoner's dilemma [2], multi-fractal properties of Wolfram's rule 90 [3], and so on.

We here propose semi-analog CAs based on Wolfram's rules 90 and 150 having historic memory capabilities. Original difference equations of 1-D CAs with rules 90 and 150 are given by

$$x_i^{t+1} = f(x_{i-1}^t, x_i^t, x_{i+1}^t), \tag{1}$$

where x_i^t represents the i-th cells state ($\in$ 0, 1) at time t, $f(\cdot)$ the exclusive OR function defined by $x_{i-1}^t \oplus x_{i+1}^t$ and $x_{i-1}^t \oplus x_i^t \oplus x_{i+1}^t$ for rules 90 and 150, respectively. We rewrote the binary difference equation into analog difference equations to include historic memory terms. Let us start from a leaky integrator model given by $\dot{x_i} = -\alpha x_i + f(\cdot)$ ($\alpha \geq 0$) that can be approximated as $x_i^{t+\Delta t} = (1 - \Delta t\alpha)x_i^t + f(\cdot)$ ($\Delta t \ll 1$). When $\Delta t = 1$, we obtain

$$x_i^{(t+1)} = (1-\alpha)x_i^t + f(x_{i-1}^t, x_i^t, x_{i+1}^t), \tag{2}$$

which represents our analog CA dynamics with historic memories. When $\alpha = 1$ the equation above exactly equals to Eq. (1), whereas for $\alpha < 1$, the historic memory effect appears, i.e., the past cell's states are preserved by the leaky integrator as analog values. Because of this analog expansion, the activate functions $f(\cdot)$ must also be expanded to continuous analog functions. We thus defined the following two functions:

$$f(x) = \begin{cases} \theta(x-1/2) - \theta(x-3/2) & , \ x \equiv x_{i-1}^t + x_{i+1}^t \text{ (Rule 90)} \\ \theta(x-1/2) - \theta(x-3/2) + \theta(x-5/2) , & x \equiv x_{i-1}^t + x_i^t + x_{i+1}^t \text{ (Rule 150)} \end{cases},$$

where $\theta(x)$ is the sigmoid function given by $[1 + \exp(-\beta x)]^{-1}$. Figure 1 plots these functions with several values of β. Notice that when $\beta \to \infty$ and $\mathbf{x} \in 0, 1$, these functions correspond to the exclusive OR functions, e.g., in Rule 90, $x_{i-1} \oplus x_{i+1}$ = logical "1" only when $x_{i-1} + x_{i+1} = 1$.

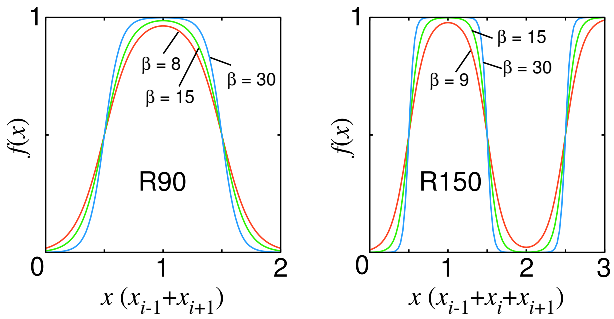

Figure 1: Activate functions of analog CA based on rules 90 (left) and 150 (right).

We conducted numerical simulations of Eq. (2) with 100 cells. Figure 2 shows the evolution of the patterns. The initial cell states were $x_{50} = 1$ and $x_i = 0$ ($i \neq 50$). The results showed that i) the analog CAs generated self-similar patterns both in the rules as conventional CAs when the historic memory effect was attenuated ($\alpha = 1$), whereas ii) dense spatial patterns were generated while maintaining the self-similar properties by historic memories when $\alpha < 1$.

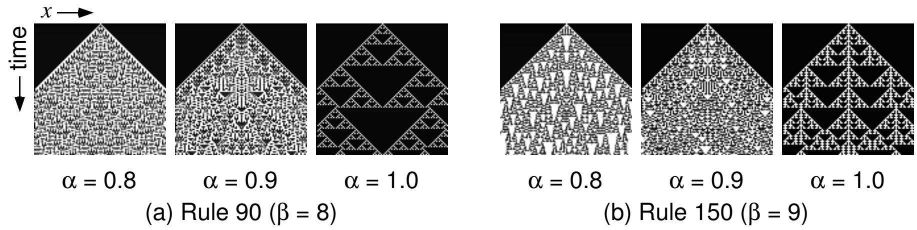

Figure 2: Pattern evolution of analog CA with rules 90 (left) and 150 (right).

To evaluate the performance of our analog CAs for random number generation, we defined the following two variables:

$$X^t \equiv \sum_{i=0}^{N/2-1} 2^{i-\frac{N}{2}} \cdot H(x_i^t - 1/2), \qquad Y^t \equiv \sum_{i=0}^{N/2-1} 2^{i-\frac{N}{2}} \cdot H(x_{i+N/2}^t - 1/2),$$

where $H(\cdot)$ represents the step function. Figure 3 shows X^t-Y^t plots (10,000 updates) with the same initial conditions as in Fig. 2 and random initial conditions. Statistically better random numbers were generated when $\alpha = 0.9$, as compared with memoryless cases ($\alpha = 1.0$).

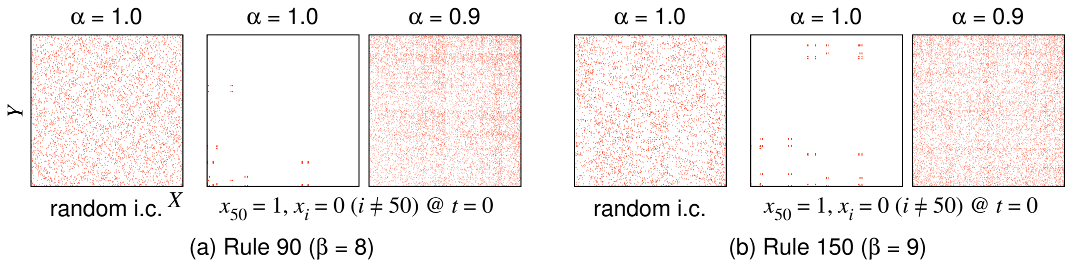

Figure 3: Random numbers generated by analog CA with rules 90 (left) and 150 (right).

Finally we designed analog electrical circuits for CMOS LSIs implementing the proposed historic analog CAs, aiming at the applications to the random-number generation. Figure 4 shows the construction of the cell circuit. We demonstrate the operations by both circuit simulations (SPICE) and experiments using discrete semiconductor devices.

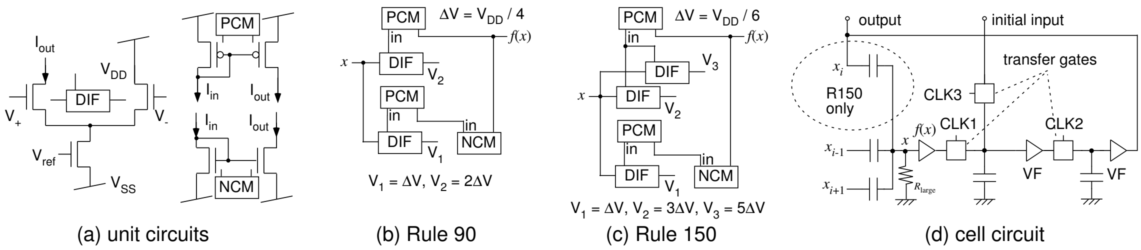

Figure 4: Circuit implementation of analog CA with rules 90 and 150 function units.

References

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