Reaction-Diffusion Chip based on Cellular-Automaton Processing

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***Abstract*—** A novel silicon LSI chip is proposed for producing the lively, dynamic behavior of reaction-diffusion (RD) systems. The RD chip consists of a two-dimensional array of cell circuits and simulates the Belousov-Zhabotinsky reaction in cellular automaton processing. Spatiotemporal pattern-generation of the RD chip was demonstrated using the simulation program with integrated circuit emphasis (SPICE).

I. Introduction

The reaction-diffusion (RD) system is a complex, dynamic system in which the reaction and diffusion of chemical species coexist under a nonequilibrium condition[1]. It produces a variety of the orders, rhythms, and self-organizing phenomena observed in nature. Implementation of the RD system in hardwares will provide practical devices that implement various ideas for information processing, e.g., ideas for chemical image processing[2, 3], optimal path planning[4], and binary logic processing[5].

Aiming at the development of such applications, our group has proposed several LSIs, including an RD chip that restores fingerprint images[6] and an RD neurochip for cortical information processing[7]. In this paper, we propose a novel RD chip that imitates the Belousov-Zhabotinsky (BZ) reaction. As the first step toward the development of RD-based applications, we demonstrated edge-detection processing on the chip by using a simulation program with integrated circuit emphasis (SPICE).

II. Chip structure

The concept of the proposed RD chip is illustrated in Fig. 1. The chip is an electric analog of chemical RD systems and produces, in the form of a potential wave on the chip surface, various spatiotemporal patterns such as concentric circules, spirals, and curlicues. The chip consists of reaction cells that emulate elementary interactions between chemical substances and diffusion devices that imitate chemical diffusion of the substances. Self- and mutual-reactions between the chemical substances (U and V in Fig. 1) are implemented in the reaction cells. The cells are arranged on a hexagonal grid and are connected with adjacent cells through the diffusion devices.

The RD chip is constructed to imitate the desired BZ reaction by choosing an appropriate BZ-reaction model. One well-known model that accounts for the BZ reaction is the Oregonator[1]. Figure 2 shows the phase space of the two-variable (U and V) Oregonator with typical parameter-values. The model equations of the Oregonator are contained in the functions f and g. Depending on the parameter values, the Oregonator exhibits oscillatory [Fig. 2(a)] or excitatory behavior [Fig. 2(b)]. A cell's stability and phase are determined by the states of its adjacent cells.

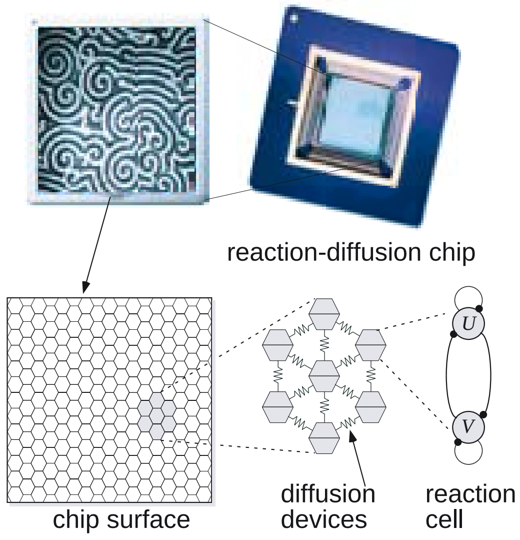
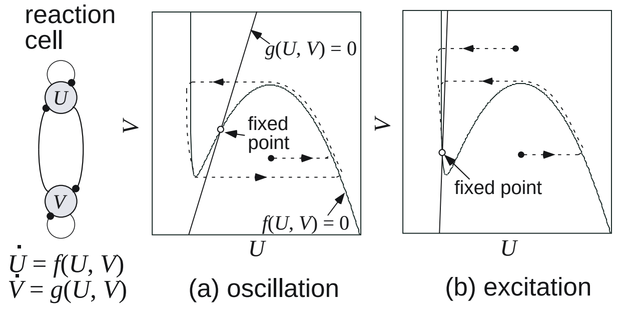


Figure 1: The reaction-diffusion chip.

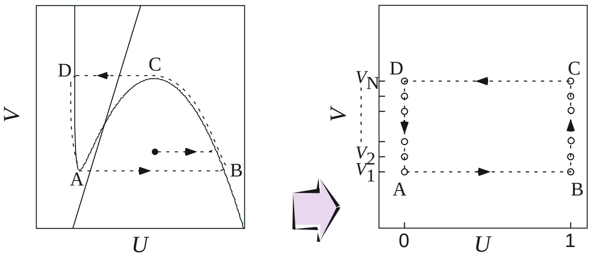
Gerhardt *et al.* proposed a simplified model of the BZ system that uses the cellular automaton (CA) method[8]. The Oregonator is usually described by continuous system-variables [Fig. 3(a)], but they introduced discrete system-variables and discrete time into the CA model. Three circulative periods are introduced at each point in the cell space: inactive, active, and refractory periods. These periods are represented by the cell state, which is expressed by excitatory variable U and inhibitory variable V. In this model, U takes a binary value representing the active period (U = 1) or the inactive one (U = 0) of the cell, while V takes multiple values ($V_1, V_2, \ldots, V_N$) representing the degree of refractory condition [Fig. 3(b)].

We embodied the CA model by arranging the cell cir-



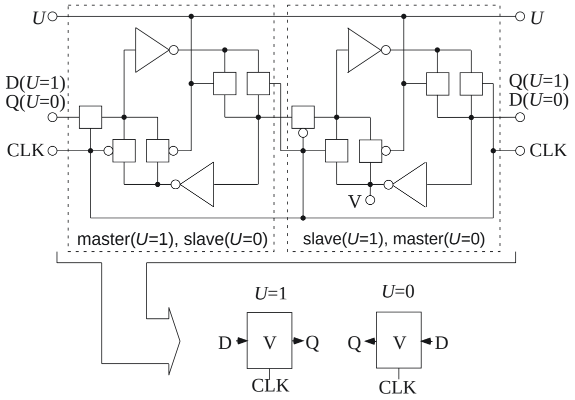
(a) oscillation (b) excitation

Figure 2: Operation modes of the Oregonator for the BZ reaction.

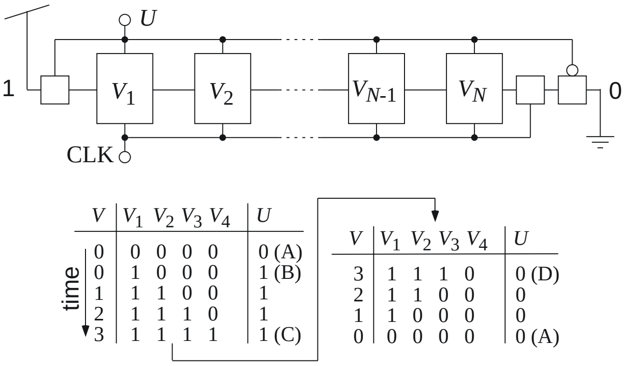


(a) continuous model (b) discrete model

Figure 3: Discrete (CA) model of the Oregonator.



(a) up-down shift register



V	V_1	V_2	V_3	V_4	U
0	0	0	0	0	0 (A)
0	1	0	0	0	1 (B)
1	1	1	0	0	1
2	1	1	1	0	1
3	1	1	1	1	1 (C)

V	V_1	V_2	V_3	V_4	U
3	1	1	1	0	0 (D)
2	1	1	0	0	0
1	1	0	0	0	0
0	0	0	0	0	0 (A)

(b) up-down shift register (N = 4)

Figure 4: Up-down shift-register circuit.

cuits on a two-dimensional hexagonal grid (see Fig. 1). In this structure, the states of the cells are synchronously updated in a sequence of discrete time steps according to a given transition rule. The rule is described in terms of the states of a cell and of its six adjacent cells. Figure 3(b) illustrates the circulative operations between the cell states. An inactive cell (A) is activated by its adjacent cell based on a given rule (A → B). It returns to the inactive state through refractory condition (C → D → A).

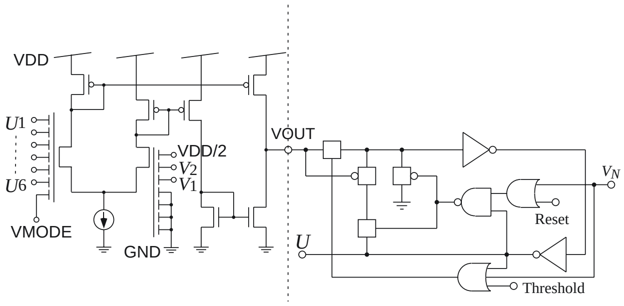
Using the CA model, we propose an analog-digital hybrid circuit for implementing the BZ operation. This cell circuit consists of an up-down (UD) shift register (Fig. 4) and a transition-decision (TD) circuit (Fig. 5). The values of the inhibitory variable (V) in the CA model are stored in the N-bit UD shift registers ($V_1, V_2, \ldots, V_N$), while those of the excitatory variable (U) are stored in the TD circuit as binary values. The up- or down-operation of the shift register is determined by the excitatory values.

Figure 5(a) shows the part of the TD circuit that compares the cell's states (V_1 and V_2) with those of its adjacent cells (U^1 to U^6) by analog operations. The circuit consists of a νMOS differential amplifier acting as a multi-input (and thus variable-threshold) comparator. The use of νMOS transistors make the cell circuit very compact compared with ordinary logic circuits providing the same functions[9].

Figure 5(b) shows the other part of the TD circuit that receives the output of the comparator (VOUT). It stores the cell state (U) according to the given transition rule. The transition rule, which is the CA parameter that determines the behavior of the BZ model, is determined by voltage "VMODE" in the comparator and voltage "THRESHOLD" connected to terminal V_1 or V_2 in the UD shift register within the same cell. Table 1 shows example transition rules (N = 4) given to the cell circuit by changing parameters "VMODE" and "THRESHOLD". The item values (0, 1, and 2) indicate the number of active adjacent cells (U = 1) required for cell transition. The cell circuit exhibits excitatory or oscillatory behavior according to the transition rule. In the excitatory mode, a cell becomes active ($U = 0 \rightarrow 1$) if one or more adjacent cells are active, then U returns to 0 after a few iterations. The number of the iteration is determined by the number of active adjacent cells. In the oscillatory mode, the cell exhibits spontaneous oscillatory behavior ($U = 0 \rightarrow 1 \rightarrow 0 \rightarrow \ldots$) with a positive phase shift generated by its active adjacent cells.

III. Simulation Results

To validate the operation of our proposed RD chip, we designed a circuit consisting of 50 by 50 cells and tested its operation by SPICE simulations, assuming a 0.6-μm



(a) νMOS comparator (b) cell-state memory

Figure 5: Transition-decision circuit.

VMODE = 0

V	Threshold V_1	Threshold V_2
3	x	x
2	x	x
1	x	2
0	1	1

(a) Excitation

VMODE = 1

V	Threshold V_1	Threshold V_2
3	x	x
2	x	x
1	x	1
0	0	0

(b) Oscillation

Table 1: Transition rules for (a) excitatory- and (b) oscillatory- operation modes.

double-poly CMOS process.

Figure 6 shows an example operation of a chip in excitatory mode (N = 3, "VMODE" = logical "0", "THRESHOLD" connected to V_1). Each cell state is represented in grayscale (V = 0: black, V = 1: gray, V = 2: white). In the circuit's initial state, cells adjacent to inactive cells were in a refractory period (step 0 in Fig. 6). The inactive cells adjacent to the white bar in Fig. 6 were inhibited by adjacent cells in the refractory period (cells in the white bar). The inactive cells then entered an active, inactive, or refractory period, depending on the degree of inhibition. When the inactive cells were in an active or inactive period, the tip of the bar rotated inward, resulting in the generation of the spiral patterns typically observed in the BZ reaction[10] (steps 2 to 8). Hexagonal distortion of the propagating waves was generated by interactions between adjacent cells.

Figure 7 shows an example operation of a chip in oscillatory mode (N = 4, "VMODE" = logical "1", "THRESHOLD" connected to V_2). The diamond pattern was given to the circuit as an initial state (step 0). The initial pattern become inverted after two iterations (step 2); the edges of the initial pattern were then extracted after two more iterations (step 4). The same phenomena have been observed in the BZ reaction[2]. Note that the underlying mechanism of this operation (edge extraction) in the BZ reaction and in our RD chip is completely different from that of ordinary digital computers.

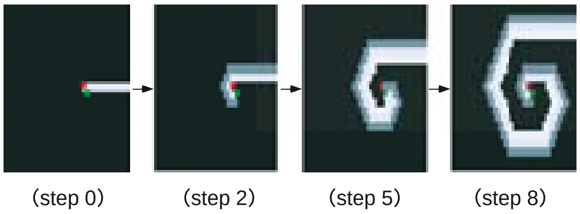


Figure 6: Excitatory operations of the RD chip.

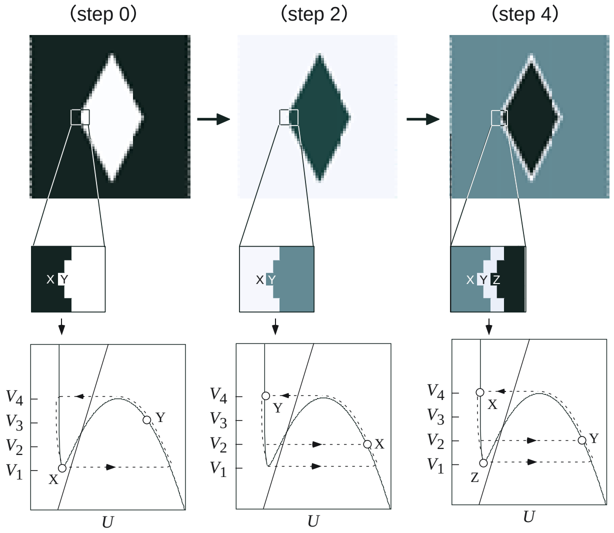


Figure 7: Oscillatory operations of the RD chip.

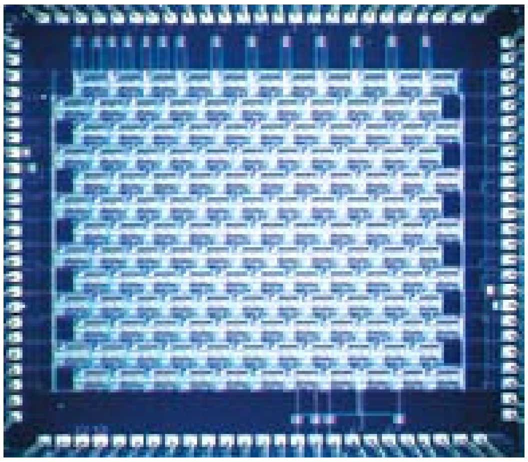
We designed the RD chip using the 0.6-μm double-poly triple-metal CMOS process. Figure 8(a) shows a micrograph of a fabricated chip implementing 11 by 13 cell circuits on 4.9-mm^2 silicon die. The unit cell circuit [Fig. 8(b)] occupies an area 210 by 170 μm. This area can be drastically reduced by using a process for laying out νMOS transistors.

IV. Conclusion

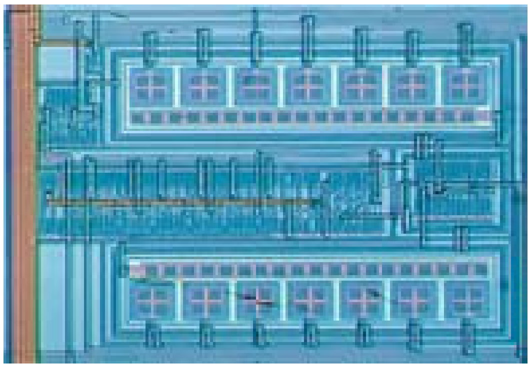
We developed a reaction-diffusion (RD) chip based on cellular-automaton (CA) processing that mimics the Belousov-Zhabotinsky (BZ) reaction.

The proposed chip has the following features.

- The chip consists of a number of identical cell circuits regularly arrayed on its surface. The cells change their states synchronously in discrete time steps according to a given transition rule. The state transition of each cell represents the chemical reaction at a point in the BZ system. The chip thus imi-



(a) chip micrograph



(b) unit cell circuit

Figure 8: Chip micrograph of the fabricated RD chip.

tates the RD dynamics of a two-dimensional BZ reaction system.

- Each unit cell consists of a hybrid structure of digital subcircuits (cell-state memory) and νMOS analog subcircuits for determining subsequent states of the cell. The use of this structure makes the cell circuit compact.
- At any moment, the operation of the chip can be stopped and the state of each cell can be retrieved.
- The RD parameters can be adjusted over a wide range by using external control signals. The use of this chip will make possible the creation of novel RD dynamics that never exist in the natural world.

The SPICE simulation showed that the proposed chip can produce typical spatiotemporal patterns observed in the BZ reactions. These results indicate that the proposed RD chip can be easily integrated into existing digital systems and can be used to clarify RD systems, aiming at developing further novel applications.

Acknowledgement

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