A Single-Electron Device for Computational Geometry
—Constructing the Voronoi Diagram by Means of Single-Electron Circuits—

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Introduction

Computational geometry is the study of efficient algorithms for solving geometric problems. It is an advancing area in the field of theoretical computer science. Combining the concept of computational geometry with actual electronic devices will enable us to open up a new field in electronics. In this paper, we propose a single-electron device that solves a problem of computational geometry — the construction of the Voronoi diagram.

Voronoi diagram

The Voronoi diagram, for a set of object points on a plane, is a partition of the space into cells, each of which consists of points that are closer to one object points than to any other object points. Figure 1 shows three examples of the two-dimensional Voronoi diagram. The Voronoi diagram has been constructed by means of mathematical methods such as the Fortune's sweep-line algorithm based on divide-and-conquer approach. Recently, Costello and others proposed a novel, non-mathematical method that used chemical systems, reaction-diffusion (RD) systems, to construct the Voronoi diagram [1]. The method uses the behavior of nonlinear waves excited in RD systems. A nonlinear wave starts at each object point (Fig. 2 (a)), propagates in all directions to form an expanding circular pattern (Fig. 2 (b)), and collide with waves from other object points (Figs. 2 (c) and (d)). The front lines of the collision make the Voronoi diagram for the object points (Fig. 2 (e)).

Single-electron device for computing the Voronoi diagrams

To apply the algorithm to electric devices, we propose using single-electron reaction-diffusion devices that we have proposed. The RD device consists of arrayed single-electron oscillators and can imitate the operations of chemical RD systems (see [2]). Figure 3 shows a configuration of single-electron RD devices that we modified for the construction of the Voronoi diagram. The device consists of arrayed single-electron oscillator with coupling capacitors (C_L) and resistances (R_d). The oscillator consists of a multiple-tunneling junction (C_m), a minute dot, resistance (R_i), and a bias voltage source (V_{dd}). In this device, a nonlinear voltage wave is excited at a point with a triggering signal. The wave propagates to form a circular pattern and stops traveling just before it collides with other waves.

Simulation results

We confirmed by computer simulation that the proposed device produced a partitioning line as in the Costello's chemical RD system. Figure 4 shows a result for a device with 100 × 100 oscillators: the waves of dot potential started at object points and traveled in all directions (Fig. 4 (a)-(d)). Then the waves stopped their traveling just before collision with each other (Fig. 4 (e)). From the data of the boundary points ("A" in Fig. 4 (e)), we were able to construct the Voronoi diagram.

References

[1] B. De Lacy Costello and A. Adamatzky, *Int. J. Bifurcation and Chaos*, Vol. 13, No. 2, pp. 521-533, 2003.

[2] T. Oya, T. Asai, T. Fukui, and Y. Amemiya, *Int. J. Unconventional Computing*, Vol. 1, No.2, pp. 177-194, 2005.

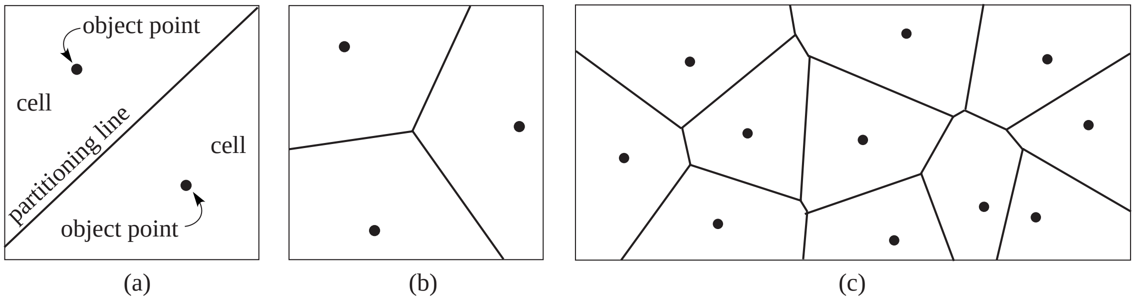

Fig. 1 Examples of two-dimensional Voronoi diagrams of (a) two, (b) three and (c) many object points.

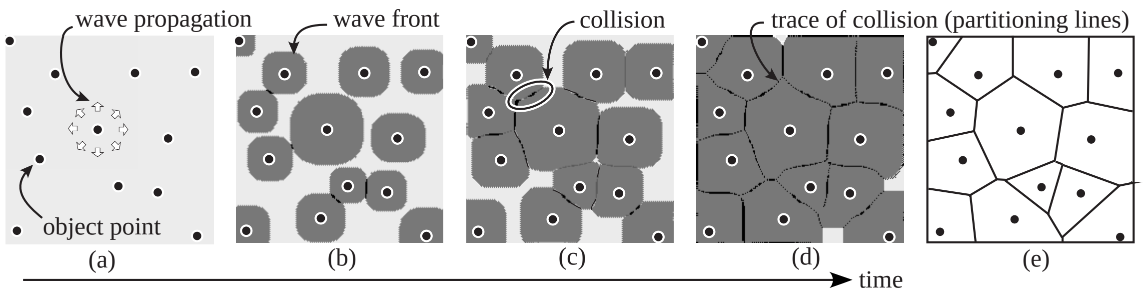

Fig. 2 Construction of the Voronoi diagram by means of a reaction-diffusion system. Wave fronts are shown in bright color, wave tracks are gray, and traces of collision are black. In this figure, the traces describe partitioning lines for the Voronoi diagrams.

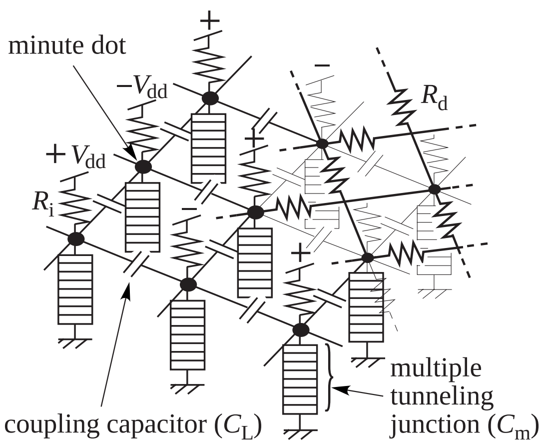

Fig. 3 Two-dimensional single-electron RD device for constructing the Voronoi diagram. The device consists of arrayed single-electron oscillators with coupling capacitors (C_L) and resisters (R_d). The oscillator consists of multiple tunneling junction (C_m), minute dot, resistance (R_i), and bias voltage source (V_{dd}). C_L connects the oscillator to other one that is biased by opposite polarity of V_{dd}. In contrast, R_d connects oscillator to other one that is biased by same polarity of V_{dd}.

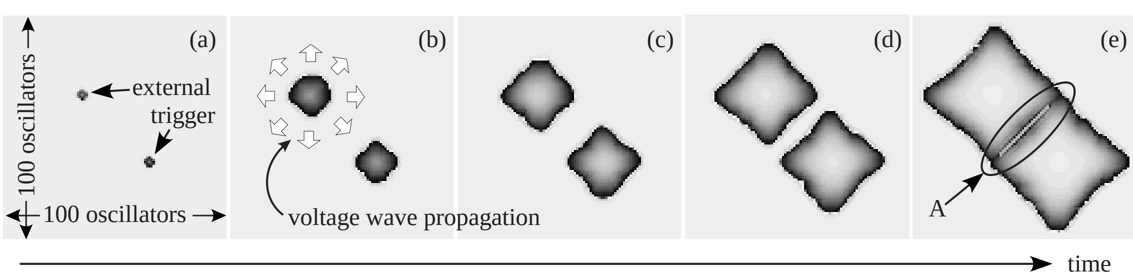

Fig. 4 Operation of the proposed RD device (simulation). The voltage wave propagates on the device ((a)-(d)), and stops traveling just before they collide with other one (see a marked area of "A" in (e)). The pattern is represented with a grayscale: a bright color means a high dot voltage, and a dark means a low dot voltage.