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Quantum Hopfield Network Using Single-Electron Circuits—A Novel Hopfield Network Free from the Local-Minimum Difficulty

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Abstract. The concept of the quantum Hopfield network is proposed with examples of its network construction, which uses single-electron circuits. In this network, two or more threshold elements can change their outputs simultaneously in a form of coherent combination. This can be put into physical form by utilizing the co-tunneling phenomenon found in single-electron circuits. In the quantum Hopfield network, a state transition with a large Hamming distance can occur and therefore the local-minimum difficulty disappears; in consequence the global-minimum energy state can always be achieved. Use of this property made possible the development of novel computation devices that solve combinatorial problems without hindrance from the local-minimum difficulty.

Key Words: Hopfield network, single electron, circuit, local minimum

1. Introduction

We here propose the concept of the *quantum Hopfield network*, a quantum version of classical Hopfield networks. The quantum Hopfield network has the possibility of providing an efficient way of solving various combinatorial problems without hindrance from the local-minimum difficulty. We also propose a way of putting this concept into physical form that utilizes the co-tunneling phenomenon in single-electron circuits.

Introducing quantum mechanics into computation may produce the capability for massive parallel processing. The quantum generalization of the Turing machine, known as the quantum Turing machine [1], is an example. The quantum Turing machine can perform ultrahigh-speed computation because it can accept as input a coherent superposition of many different data and subsequently perform a computation on all of these input data simultaneously. This concurrency or parallelism can be used to quickly solve several problems that are difficult with the classical Turing machine (and existing computers), such as *factoring* and *discrete logarithms*. Several approaches have been proposed for implementing the quantum Turing machine [2–4].

Is this type of quantum effect exclusive to the Turing machine? The authors do not think so. Various other computation models besides the Turing machine are known, and it is likely that the parallelism of each of them can be enhanced with the application of quantum mechanics. This paper takes the *Hopfield network* as an example and shows that quantum parallelism can be obtained in this computation models as well.

In the following sections, we will first outline the concept of the Hopfield network. It is a computation model for solving combinatorial problems that has great potential for providing the efficient solution to the problem. But in practice, the Hopfield network has remained obscure because it cannot operate as desired, owing to a bothersome phenomenon known as the *local-minimum difficulty* (Section 2). To overcome this obstacle, we then present the idea that the local-minimum difficulty can be removed by introducing quantum mechanics into the network operation. This can be achieved by utilizing a quantum effect, the co-tunneling phenomenon, in single-electron circuits. A method of constructing the network with single-electron circuits will be presented (Section 3). To demonstrate the merit of this quantum version of classical Hopfield networks, we simulated the operation of problem solving by computer calculation. Using these results, we will show that the quantum Hopfield network can solve a given combinatorial problem without interference from the

local-minimum difficulty (Section 4). Finally, we will conclude by discussing the parallelism in computation of the quantum Hopfield network (Section 5).

2. The Hopfield Network – A Computation Model for Solving Combinatorial Problems

The Hopfield network is a computation model for solving combinatorial optimization problems that employs the operation of a specific recurrent network (hereafter we call the recurrent network itself a Hopfield network.) The concept of a Hopfield network is illustrated in Fig. 1. The network consists of threshold elements and connections. The connection weights Wij and θi can be given any desired value, with the restrictions that $Wij = Wji$ and $Wii = 0$. The outputs Vi of the threshold elements i wrap around to become the inputs to the network. Each threshold element i produces an output "1" if the weighted sum of inputs $(\Sigma WijXi + \theta i)$ is positive and an output "0" if the weighted sum of inputs is negative. The point of this network is that, starting at a given initial position, it changes its *internal state* (a set of the outputs Vi of the threshold elements) to minimize the value of the *energy function* defined by

$$E = -1/2\,\Sigma WijViVj - \Sigma\theta iVi \qquad (1)$$

By adjusting the connection weights we can relate the energy function of the network to the cost function of a given optimization problem. In this way, we can find the solution to the problem simply by observing the final state that the network reaches. For details, see [5,6].

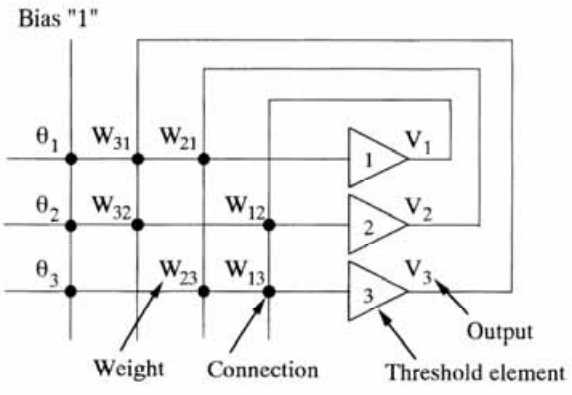

Fig. 1. Concept of the Hopfield network.

The computation in the Hopfield network is quite different from the commonly used digital computation. In the digital one, we cannot obtain the solution to a problem until we have examined all the possible combinations of the problem variables; in consequence, the computing time required increases exponentially with the size of the problem. In contrast, in the Hopfield network, a given problem is mapped onto the network itself and is solved quickly through concurrent or parallel operation of all the elements in the network. The Hopfield network, therefore, has the possibility of solving combinatorial problems in a short time regardless of the size of the problem. This parallelism may provide an efficient way of solving difficult combinatorial problems such as NP-complete (nondeterministic polynomial-time complete) problems, which are often encountered in engineering fields but take enormous computing time to solve using digital computers.

Unfortunately, it is not possible to be certain that the correct solution can always be obtained. This is because the Hopfield network in general has many states of locally minimum energy in addition to the globally minimum state. In most cases the network will get stuck in a local minimum and a solution will not be reached. The present computation model is based on the premise that the final state of the network can be considered as minimum in energy, and without this premise we cannot be convinced that an obtained result is the correct solution. This is an inevitable drawback in the Hopfield network and has limited the application field of the Hopfield network.

The local-minimum difficulty above is a natural result of the fact that each event of state transition in the threshold elements is independent of others. The threshold elements update their output states irrelevantly with no mutual correlation, and consequently the network can make at one time only a limited state transition of a *Hamming distance of 1* (i.e., a transition corresponding to the output change of one threshold element). Under these conditions the network cannot escape from a local minimum, even if there are other possible states with lower energy, because an output change of any one threshold element will increase the network energy. This is inevitable as far as we are tied to the classical concept of the Hopfield network.

The way of overcoming this difficulty is to create a special network in which *two or more threshold elements can change their outputs simultaneously in a form of coherent combination*. In such a network, the

transition of a larger Hamming distance (a value of 2 or more) can occur, and in consequence the global minimum state can be achieved without hindrance by local minima. To materialize such networks, we will consider utilizing a quantum phenomenon in single-electron circuits.

3. Constructing the Hopfield Network Using Single-Electron Circuits

The single-electron circuit changes its state to decrease its free energy. We can make use of this property to design a Hopfield network (see [7,8]). We propose here a likely circuit design, as illustrated in Fig. 2. A tunnel junction with an excess electron is used as a threshold element (Fig. 2(a)); we define the state of the tunnel junction as "0" if the electron is on the left of the junction and as "1" if it is on the right.

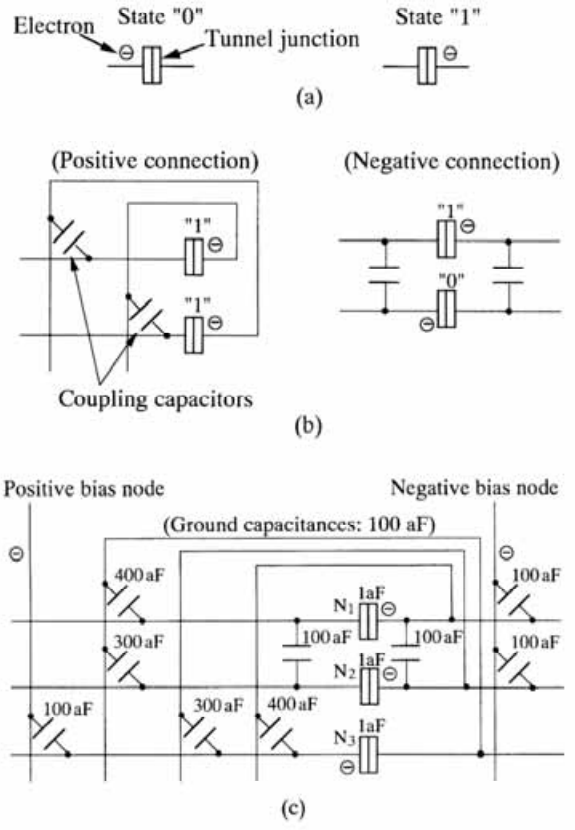

Fig. 2. Single-electron Hopfield network. (a) Tunnel junction as a threshold element. (b) Positive and negative connections. (c) A sample configuration of the network.

The connection between two tunnel junctions can be established by a pair of coupling capacitors (Fig. 2(b)); the connection weight can be set to either positive or negative, depending on the layout of the capacitor coupling. The overall configuration of the network is illustrated in Fig. 2(c). (An excess electron is also set on each bias node.) A ground capacitance exists between each node and ground (not illustrated here for simplicity). A sample set of capacitance parameters is given in the figure. Starting from a given initial position, the circuit changes its internal state (the arrangement of electrons) to minimize its free energy. We have confirmed that the free energy for this circuit is given by the following expression (see Appendix):

$$E = A - 1/2\,\Sigma BijNiNj - \Sigma CiNi \qquad (2)$$

where Ni is the state of each tunnel junction (either 1 or 0), and that the coefficients A, Bij, and Ci can be set at desired value, based on the connection pattern and the capacitance values of the tunnel junctions, connection capacitors, and ground capacitors. This equation is in essence the same as the energy function (equation (1)) for the Hopfield network. In this way we can be certain that the proposed circuit will operate as a complete Hopfield network.

The internal state of this circuit is expressed by a set of the states of the tunnel junctions. For the sample circuit in Fig. 2(c), the internal state is expressed as $(N1, N2, N3)$. The energy values for all possible internal states are calculated and plotted in Fig. 3. To show the relationship between the states, we alter Fig. 3 into the diagram illustrated in Fig. 4. The global minimum is state $(0, 0, 0)$. The solid arrows in the figure indicate the possible occurrence of a state transition due to one tunneling event, corresponding to the transition of a *Hamming distance of 1*, which also occurs in classical Hopfield networks. (We here assume zero temperatures and therefore no energy excitation.) For such transitions, states $(1, 0, 1)$ and $(0, 1, 1)$ have several incoming paths but no outgoing path; therefore these two states seem to be local minima.

But, as described in the following section, the single-electron Hopfield network has quantum properties unlike its classical relatives. Consequently the states $(1, 0, 1)$ and $(0, 1, 1)$ do *not* act as a local minimum state and the network *never* becomes stuck in these states.

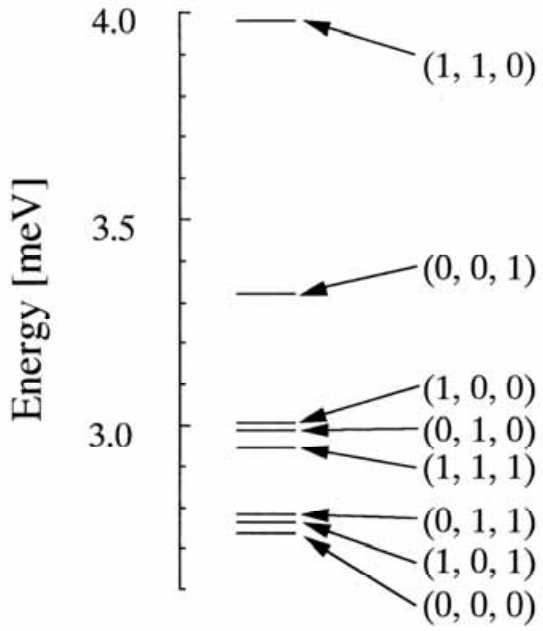

Fig. 3. Energy levels for the possible states in the sample network of Fig. 2(c).

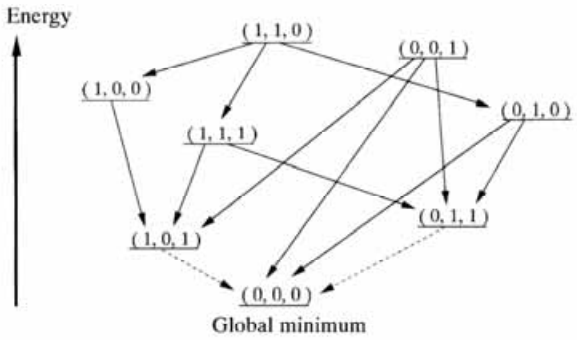

Fig. 4. Energy diagram showing possible transitions for the sample network of Fig. 3(c). A solid arrow shows a transition by one tunneling event (Hamming distance of 1), while a dashed arrow shows a transition by co-tunneling of two tunneling events (Hamming distance of 2).

4. Quantum Operation in Single-Electron Hopfield Networks

The dashed arrows in Fig. 4 indicate the state transition of a *Hamming distance of 2*, which can occur only when two threshold elements (tunnel junctions) change their states simultaneously with mutual correlation. Such a transition is nonexistent in a classical sense, but in the single-electron Hopfield network, it can actually occur through a quantum effect known as the *co-tunneling phenomenon*.

Co-tunneling is a phenomenon in which two or more tunneling events occur simultaneously in a form of quantum coherent combination. In single-electron circuits, two or more tunnelings can occur simultaneously through co-tunneling if such an event decreases the energy of the circuit. This makes possible transitions of a larger Hamming distance (2 or more) in single-electron Hopfield networks. Through this phenomenon, the sample circuit in Fig. 2(c), for example, can change its state from $(1, 0, 1)$ and $(0, 1, 1)$ to the global minimum $(0, 0, 0)$, as illustrated by the dashed arrows in the figure; thus the local-minimum difficulty disappears. We call this type of Hopfield network a *quantum Hopfield network*. In the quantum Hopfield network, it is certain that, starting at a given initial state, the global minimum state can always be established.

To see the behavior of the single-electron quantum Hopfield network, we here observe through computer simulation the state transition in the single-electron network circuits. In the simulation, a Monte Carlo method was used that is combined with the basic equations for electric-charge distribution, charging energy, and tunneling probability; the probabilistic characteristic of electron tunneling is introduced through the use of random numbers (see [9]). The co-tunneling phenomenon is taken into account by the method in [10]. The temperature was assumed to be 0 K.

The result is illustrated in Fig. 5 for the sample circuit of Fig. 2(c), using the device parameters shown in the figure (the tunnel resistance was set at 200 kΩ). The circuit was initially set at maximum energy state $(1, 1, 0)$, then was allowed to change its state without restraint. After some transition time the circuit stabilized in a final state. This procedure, a *trial*, was repeated many times using a different series of random numbers; the results of three trials are illustrated in the figure.

In every trial, observed at first were transitions of a *Hamming distance of 1* (denoted by numbers 1 through 6 in the figure). These transitions are the same ones as observed in classical Hopfield networks. The network was sometimes able to converge to minimum energy state $(0, 0, 0)$ through transitions of a *Hamming distance of 1* (as shown by numbers 2 and 5 in the figure), but usually became stuck in the intermediate states $(1, 0, 1)$ or $(0, 1, 1)$ (as represented

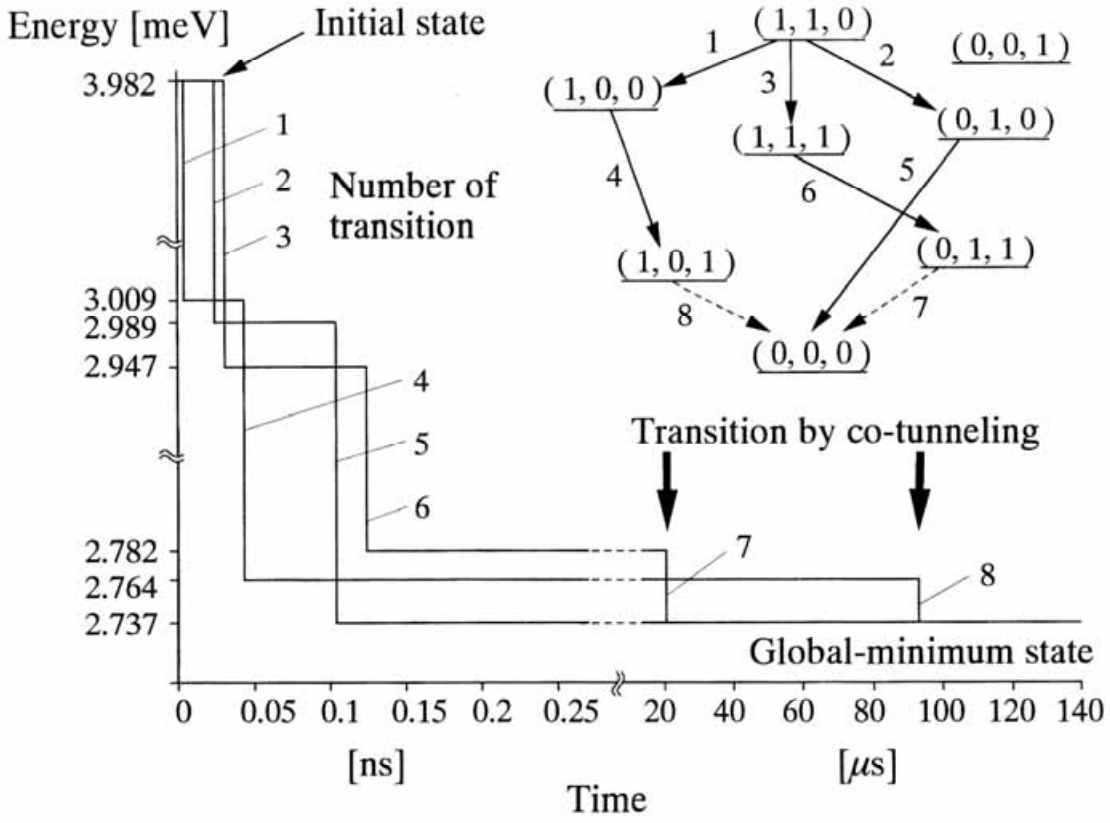

Fig. 5. State transition in the single-electron Hopfield network of Fig. 2(c) (computer simulation). The results of three trials are plotted. The circuit finally achieves the global-minimum state.

by numbers 1 and 4, or 3 and 6). If we had been dealing with a classical Hopfield network, the two states would have acted as local minima. The situation in the single-electron Hopfield network was, however, quite different. After some waiting time, we were able to observe the transition from the states $(1, 0, 1)$ or $(0, 1, 1)$ to the global minimum state $(0, 0, 0)$ (as shown by numbers 8 or 7). This transition was a transition of a *Hamming distance of 2* that was induced by the co-tunneling phenomenon.

We have confirmed, by simulation, the same quantum operation for various network samples. In complex networks, state transitions of larger Hamming distances (3 or more) are required for convergence, but after a certain amount of waiting time, these transitions are sure to occur. In general, single-electron Hopfield networks can always reach the global-minimum energy state, starting at a given initial state. Using this property will make possible the development of novel computation devices that solve combinatorial problems without being troubled by the local-minimum difficulty.

5. Summary

The concept of the quantum Hopfield network was proposed and examples were given of its network construction, which uses single-electron circuits. In this network, two or more threshold elements can change their outputs simultaneously in a form of coherent combination. This concept can be put into physical form by utilizing the co-tunneling phenomenon in single-electron circuits. In the quantum Hopfield network, a state transition of a large Hamming distance can be occurred, and in consequence the global-minimum energy state can be always achieved, without hindrance from the local-minimum difficulty. The network calculates simultaneously many energy values for all the possible

combinations of the junction states to find the minimum energy state. Thus quantum parallelism is obtained, though in a form different from that of the quantum Turing machine. The quantum Hopfield network will therefore provide an efficient computing tool for solving various combinatorial problems.

An open question is whether it would be practical to build single-electron network circuits that can generate the co-tunneling event frequently enough to deal with any given complicated problems. Although various difficulties lie ahead, the authors believe that theoretical and technological progress will sooner or later make such network devices feasible.

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Appendix

Equation (2) in the text can be arrived at as follows. The free energy in the proposed circuit is equal to electrostatic energy, so the energy value is given by summing *the charging energy stored on each capacitance* in the circuit (each of the coupling capacitances, the tunnel junction capacitances, and the ground capacitances). The free energy is therefore given by a quadratic function of *the charge on each capacitance*. Based on the principle of superposition in electric circuits, the charge on each capacitance can be given by a linear function of *the total charge on each node* in the circuit; the total charge on each node is $-eNi$ (e: the electron charge, Ni: the state of tunnel junction i) for the right-hand nodes of tunnel junction i, $-e(1-Ni)$ for the left-hand nodes of tunnel junction i, and $-e$ for the bias nodes. In consequence the free energy of the circuit can be put into the form of a quadratic function of Ni. Considering that $Ni^2 = Ni$ (because Ni is either 1 or 0), the energy function can be reduced to equation (2) given in the text.

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