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STOCHASTIC RESONANCE IN A BALANCED PAIR OF SINGLE-ELECTRON BOXES

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Stochastic resonance in a fundamental single-electron circuit, i.e., a balanced pair of single-electron boxes, is observed and presented theoretically, where the signal-to-noise ratio (SNR) of the internal states stimulated by responding to a periodic subthreshold or suprathreshold input is enhanced by thermal agitation in tunneling junctions. Through extensive Monte-Carlo simulations, the peak SNR was determined as a function of temperature and input amplitudes. These results imply the possibility to design single-electron circuits that may "exploit" thermal noise, instead of employing conventional noise-suppression strategies.

Keywords: Stochastic resonance; signal-to-noise ratio; Monte Carlo simulations; thermal noise.

1. Introduction

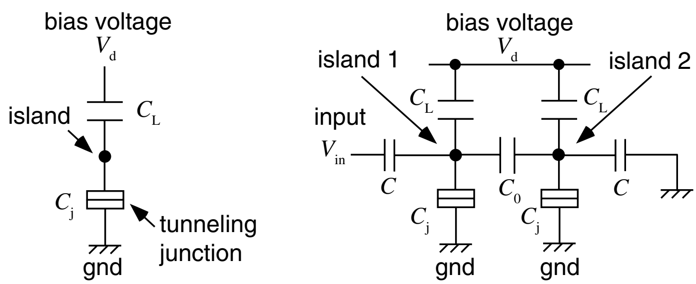
In single-electron circuits, thermal noise is usually considered an "obstacle" that induces undesirable fluctuations of the Coulomb blockade. Most strategies for dealing with them focus on suppression, e.g., by decreasing the operation temperature or the junction capacitance. *Stochastic resonance* (SR) offers a different strategy aiming at exploiting thermal noise to improve circuit performance [1]. This paper shows this approach using a balanced pair of single-electron boxes, as an example circuit.

SR is a phenomenon where a static or dynamic thresholding system responds stochastically to a subthreshold or suprathreshold input with the help of noise (see [2] for an overview). Even a simple stochastic resonator that consists of a static threshold detector does greatly improve the signal-to-noise ratio (SNR) of a periodic input with additive Gaussian noise [3]. SR has been observed in many electronic systems, e.g., static thresholding systems [1, 4, 5], bistable systems [6], laser devices [7, 8], etc. SR in a static thresholding system can be utilized for detecting weak (subthreshold) signals. Indeed, we have already demonstrated SR in a single-electron thresholding system for weak-signal detection, where a parallel array of single-electron boxes was used as thresholding elements [9]. On the other hand, SR in a bistable system may be used to enhance SNR in logic memory systems that receive insufficient input driving strength to enable data rewriting in a noisy environment [10].

This paper describes the beneficial use of thermal agitation to demonstrate SNR enhancement in single-electron bistable circuits. A balanced pair of single-electron boxes [11] (bistable circuit) is used as an example, which has not been specifically designed to present noise robustness. In Sec. 2, we briefly review the operation of the balanced pair of single-electron boxes. In Sec. 3, we evaluate the SNR versus temperature by applying subthreshold inputs to the circuit, and discuss the simulation results of the circuit driven by subthreshold and suprathreshold inputs. Section 4 is devoted to a summary.

2. Brief Review of Balanced Pair of Single-Electron Boxes

Figure 1(a) shows a single-electron box consisting of an ideal tunneling junction C_{j}, a load capacitor C_{L} and a bias voltage source V_{d}. An isolated island forming a quantum dot is placed between C_{j} and C_{L}. Electrons can only enter and leave the island through C_{j}. Increasing V_{d} will populate the island with an integer number



(a) (b)

Fig. 1. Circuit configuration of (a) single-electron box and (b) balanced pair of single-electron boxes.

of elementary excess charges ($\equiv n$) leading to a charge staircase characteristics on the island [12]. In an ideal environment (operational temperature T is set at 0 K, $C_{\mathrm{j}} = C_{\mathrm{L}}$ ($\equiv C_0$), and the island has no background charge), when V_{d} is increased from 0 V, the number of elementary excess charges is given by

$$n = \left\lfloor \frac{2C_0}{e} V_{\mathrm{d}} \right\rfloor, \tag{1}$$

where $\lfloor\cdot\rfloor$ represents the floor (staircase) function; and e, the elementary charge.

Figure 1(b) shows a balanced pair of single-electron boxes [11]. The circuit has two islands, and its internal "state" is expressed by (n_1, n_2) where n_1 and n_2 represent the numbers of excess electrons on islands 1 and 2. The circuit is bistable when

$$\frac{C_{\mathrm{j}} + C_{\mathrm{L}} + C_0 + 3C}{2C_{\mathrm{L}}(C_{\mathrm{j}} + C_{\mathrm{L}} + 2C_0 + 3C)} e < V_{\mathrm{d}} < \frac{C_{\mathrm{j}} + C_{\mathrm{L}} + 3C_0 + 3C}{2C_{\mathrm{L}}(C_{\mathrm{j}} + C_{\mathrm{L}} + 2C_0 + 3C)} e, \tag{2}$$

at $T = 0$ K and $V_{\mathrm{in}} = 0$ [11]. If $C_0 \equiv C_{\mathrm{j}} = C_{\mathrm{L}}$ and $C = 3C_0$, (2) is simplified as

$$\frac{6}{7} V_0 < V_{\mathrm{d}} < \frac{8}{7} V_0, \tag{3}$$

where $V_0 \equiv e/(2C_0)$. Hence, if V_{d} is set at V_0, the circuit's stable states are $(1, 0)$ or $(0, 1)$. When V_{in} is positive, electron tunneling at island 1 (from the ground to island 1) occurs earlier than electron tunneling at island 2 because the potential at island 1 is higher than at island 2. Consequently, the circuit becomes stable at $(1, 0)$. On the other hand, when V_{in} is negative, the circuit becomes stable at $(0, 1)$.

Figure 2 shows the transition diagram of the ideal balanced pair[a] for continuous increasing or decreasing of V_{in} from 0 V. The circuit is bistable [stable at $(0, 1)$ or

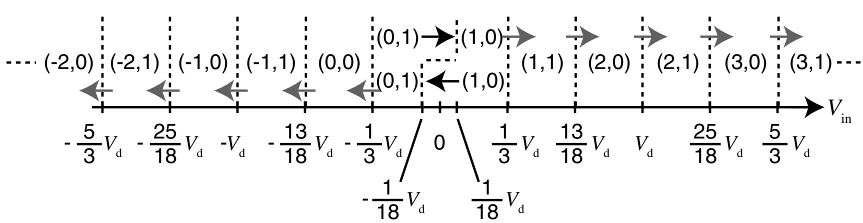


Fig. 2. Transition diagram of state (n_1, n_2) in ideal balanced pair circuit at $T = 0$ K.

[a]The threshold values for state transition were analytically calculated following the initial procedure of the Monte Carlo simulation for single-electron circuits [12]; i.e., (i) calculate charge distribution of entire circuit network for given V_{in}, V_{d}, and (n_1, n_2), (ii) assume possible state transitions (n_1', n_2) or (n_1, n_2') (electron tunneling at one island), and re-calculate the charge distribution, (iii) calculate the electrostatic energy difference between states (i) and (ii) ($\equiv E_1 - E_2$), as well as work E_3 done by V_{in} and V_{d} for the assumed electron tunneling, (iv) estimate the critical V_{in} ($\equiv V_{\mathrm{th}}$) where $E_1 - E_2 + E_3 = 0$.

$(1, 0)$] when

$$-\frac{1}{3}V_{\rm d} < V_{\rm in} < \frac{1}{3}V_{\rm d}, \tag{4}$$

and state transition between $(1, 0)$ and $(0, 1)$ occurs at

$$V_{\rm in} = \pm V_{\rm th}, \quad \text{where} \quad V_{\rm th} \equiv \frac{1}{18}V_{\rm d}. \tag{5}$$

Consequently, positive (or negative) $V_{\rm in}$ induces transition of $(0, 1) \to (1, 0)$ [or $(1, 0) \to (0, 1)$].

In the following, let us assume that the circuit accepts a periodic input, e.g., $V_{\rm in} = A_0 \sin(2\pi f t)$, where A_0 is the amplitude; f, the frequency; and t, the time. At $T = 0$ K, periodic state transition between $(1, 0)$ and $(0, 1)$ may occur when $V_{\rm th} < A_0 < V_{\rm d}/3$ (suprathreshold input with bistable constraints), whereas no transition occurs when $A_0 < V_{\rm th}$ (subthreshold input). SR predicts that a "stochastic transition" may be caused by a subthreshold input in presence of noise, which relates to temperature T in the case of single-electron circuits. Nonzero T may induce stochastic electron tunneling, and the probability of a desirable electron tunneling event is increased if $V_{\rm in}$ approaches to $\pm V_{\rm th}$. On the other hand, large T may disturb the Coulomb blockade, and may cause random transition. Therefore, we expect that there exist optimal T that maximizes the SNR of state transition.

3. Simulation Results and Discussion

Extensive Monte Carlo simulations have been conducted to confirm the SR behavior of the balanced pair of single-electron boxes. Custom software as well as the SIMON simulator [13] have been used to extract and confirm the results presented in the following. The circuit parameters are set to $C_0 = C_{\rm j} = C_{\rm L} = 1$ aF, $C = 3C_0$, $V_{\rm d} = 80.1$ mV ($\approx e/2C$), $V_{\rm th} = V_{\rm d}/18 \approx 4.4$ mV, and the input is given by $V_{\rm in} = A_0 \sin(2\pi f t)$.

In response to AC signals, the circuit operates as a RC filter. All nodes are subject to the input AC signal, and thus, monitoring raw node voltages is of low practicality, and needs further signal processing. In order to analyze the SR phenomenon, the following scalar function representing the vector state (n_1, n_2) is defined as

$$s(n_1, n_2) \equiv \operatorname{sgn}(n_1 - n_2) = \begin{cases} 1 : n_1 > n_2 \\ 0 : n_1 = n_2 \\ -1 : n_1 < n_2 \end{cases}, \tag{6}$$

where sgn(·) represent the sign function. In the following simulations, we use $s(n_1, n_2)$ to evaluate the circuit's transient status.

Figure 3 shows the time-domain simulation of the islands node charge evolution, in response to a subthreshold sinusoidal input inputs [$V_{\rm in} = A_0 \sin(2\pi f t)$, $f = 100\,$MHz, $A_0 = 4\,$mV ($< V_{\rm th}$)] for several temperature conditions. Ideally and at

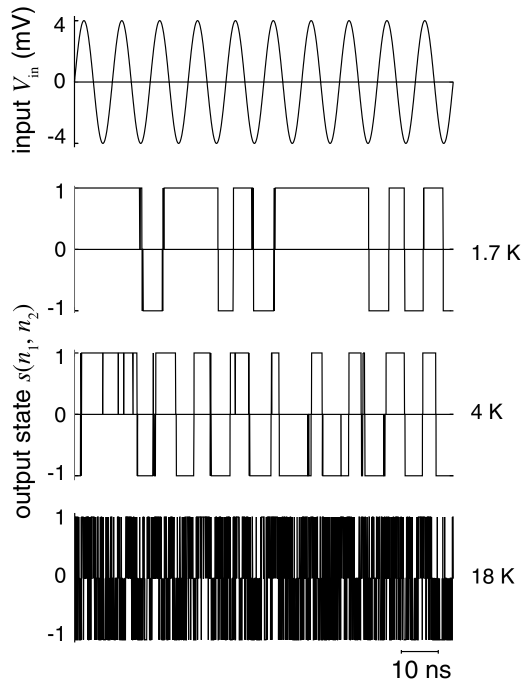


Fig. 3. Time-domain circuit output states $s(n_1, n_2)$ for subthreshold sinusoidal inputs ($V_{\text{in}} = A_0 \sin(2\pi f t)$, $V_{\text{th}} = V_{\text{d}}/18 \approx 4.4$ mV), $A_0 = 4$ mV.

$T = 0\,\text{K}$, no electron tunnels from ground to any island node upon stimulation with a subthreshold amplitude signal. At moderate temperature, e.g., $T = 1.7\,\text{K}$ and $4\,\text{K}$ in Fig. 3, thermal-induced noise tunneling enables the circuit to stochastically detect the subthreshold input, according to the following scheme:

- During the negative phase of the subthreshold input, the occurrence of an unexpected electron tunneling event causes a charge to tunnel from ground and create a charge excess on island n_2; simultaneously, the initial charge excess of n_1 tunnels to ground. The new system state is $(n_1, n_2) = (0, 1)$.
- Subsequently, during the positive phase of the subthreshold stimuli, the occurrence of an unexpected electron tunneling event causes a charge to tunnel from ground and create a charge excess on island n_1; simultaneously, the initial charge excess of n_2 tunnels to ground. The new system state is $(n_1, n_2) = (1, 0)$, i.e., has returned into its initial state.

It should be noted that the aforementioned events are assumed simultaneous on n_1 and n_2, which is an acceptable postulate, though not rigorously correct. The new system states hold until the occurrence of a new unexpected electron tunneling that causes a system change. Thus, taking benefit of thermal noise which manifests itself as unexpected electron tunneling events, the circuit acquires the ability to detect

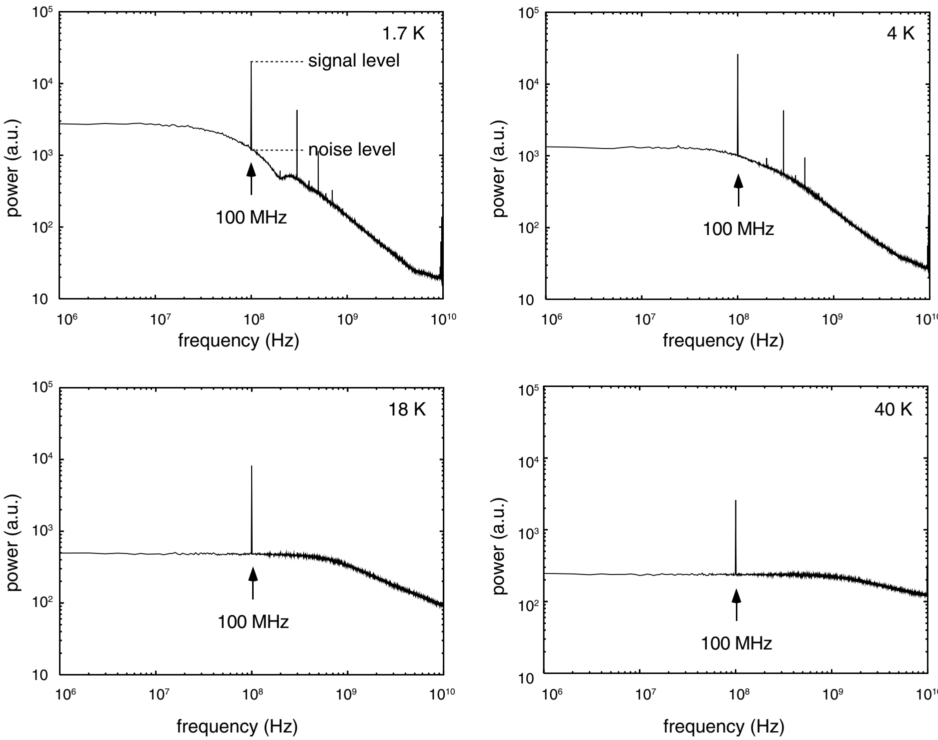


Fig. 4. Power spectrum of output states $s(n_1, n_2)$ of Fig. 3.

the subthreshold stimuli. The unexpected electron tunneling rate increases with temperature, which is evidenced by the high density of events observed in Fig. 3 at 18 K, and at elevated temperature, noise is expected to dominate the signal component. Clearly, noise improves the circuit performance at moderate temperature level and deteriorates the circuit performance at high level. The signal-to-noise ratio (SNR) is used as a figure that quantifies the improvement (or degradation) of circuit performance versus the temperature level. The power spectrum of the circuit states is obtained as the Fourier transform of $s(n_1, n_2)$, and the SNR is subsequently derived.

Figure 4 presents the power spectra of the circuit states shown in Fig. 3. The presence of a peak in the output spectrum which is at the stimulation frequency denotes the detection of the suprathreshold stimuli, and demonstrates the SR operation. Note that it is not a manifestation of the AC content of any circuit node (which is real), since only the node excess charge is used, which is obtained from quantum physics derivations, i.e., no voltage or current is involved in the signal processing.

The SNR is obtained by subtracting the background noise level at the input frequency, 100 MHz, from the signal level, in decibel. Figure 5 shows the SNR

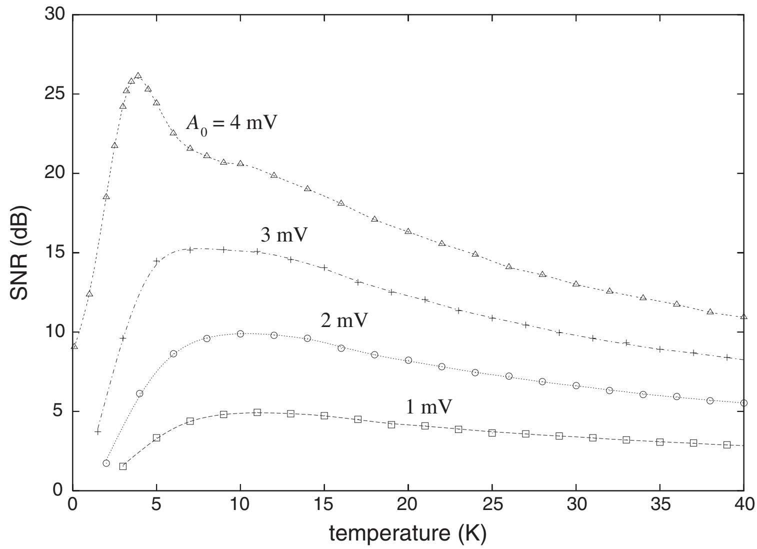


Fig. 5. SNR versus temperature at $A_0 = 1\,\text{mV}$, $2\,\text{mV}$, $3\,\text{mV}$, and $4\,\text{mV}$.

of the system over a temperature up to 40 K, and under various suprathreshold stimuli ($A_0 = 1$, 2, 3, and 4 mV). Each point is obtained as the average of thousand simulations with different random seeds. The successful SR operation is confirmed by the presence of a maxima, revealing that a functionality which is impossible at 0 K or in the range of very low and very high temperature becomes realistic within a moderated range of temperature of 4 K through 15 K, approximately. The amplitude reduction of the input stimuli causes a reduction of the observed SNR throughout the temperature range, which is observed from four different curves at $A_0 = 1$, 2, 3, and 4 mV. The maxima of the four curves are observed at 4 dB, 9 dB, 15 dB, and 26 dB, respectively.

Figure 6 shows how the SNR relates to the input signal amplitude at fixed temperature and assuming that Eq. (4) holds. A fixed temperature has been selected at $T = 10\,\text{K}$, without loss of generality. In absence of SR behavior, the circuit operation should be depicted as a staircase, where an abrupt transition separates the region of disrupted operation in the range of $A_0 = [0, V_{\text{th}}]$, i.e., SNR $\leq 0\,\text{dB}$, to the region of correct operation, in the range of $A_0 = [V_{\text{th}}, V_{\text{d}}/3]$, i.e., where the SNR is maximal. In the SR regime, a graceful degradation of the circuit performance is observed. In suprathreshold stimuli, the SNR tends to a maximum value of 62 dB with asymptotic behavior. Since temperature is fixed, the unexpected tunneling rate is also constant. However, as the stimuli amplitude is decreased close to V_{th}, larger numbers of unwanted tunneling events may cause switching of the internal states to the incorrect value. A smooth transition is observed from the regime above V_{th} to below V_{th}. Under V_{th}, performance gracefully degrades instead of exhibiting systematic

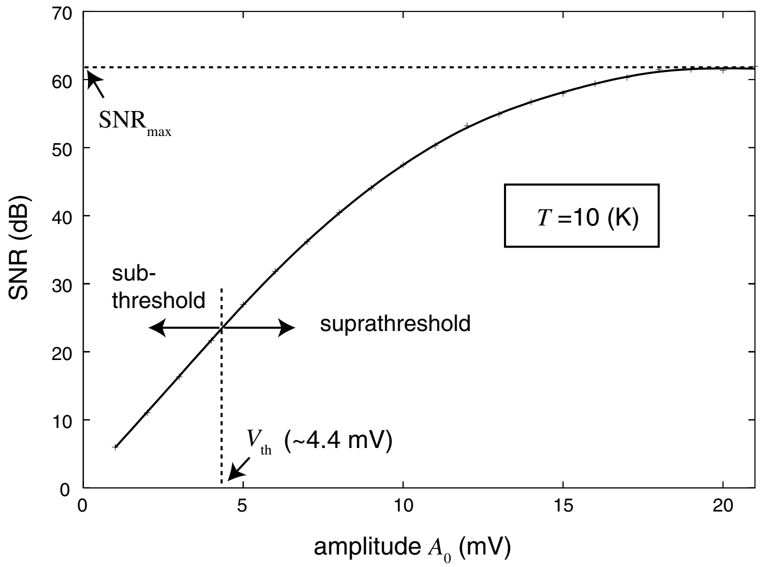


Fig. 6. SNR versus input stimuli at a fixed temperature equal to 10 K.

malfunction. Thus, operating the circuit at moderate temperature, i.e., in the range of 4 K to 15 K, approximately, is beneficial to the detection of subthreshold signal, but causes a performance degradation in conditions of weak suprathreshold signals. In summary, the circuit operates in three regimes with respect to temperature and in the presence of a suprathreshold stimuli. At low temperature, i.e., 0 K to 4 K, the signal and noise energy are insufficient to trigger a correct change of the internal circuit states. A window of optimal operation can be defined at low to moderate temperature, i.e., 4 K to 15 K, approximately, and a temperature found which optimizes the output SNR, e.g., $\mathrm{SNR}_{\mathrm{max}} = 4\,\mathrm{dB}$ at $A_0 = 1\,\mathrm{mV}$, $\mathrm{SNR}_{\mathrm{max}} = 9\,\mathrm{dB}$ at $A_0 = 2\,\mathrm{mV}$, $\mathrm{SNR}_{\mathrm{max}} = 15\,\mathrm{dB}$ at $A_0 = 3\,\mathrm{mV}$, and $\mathrm{SNR}_{\mathrm{max}} = 26\,\mathrm{dB}$ at $A_0 = 4\,\mathrm{mV}$. At high temperature, thermal noise dominates the signal by providing sufficient energy for an excessive amount of electrons to tunnel, unwantedly.

4. Summary

Theoretical developments and simulation results of the balanced pair of single-electron box operating as a stochastic resonance (SR) device are presented. Operating the balanced pair as a SR driven system shows a clear benefit in the presence of subthreshold input signals. An optimum range of operation temperature can be derived which depends on the expected amplitude of the subthreshold signal. The SR operation of the circuit shows graceful degradation of its performance, rather than an abrupt disruption. Thus, in SR regime, the balanced pair transforms into a stochastic circuit. In order to take benefit of the operation of the circuit in SR regime, its computational results must be interpreted and handled as probabilistic. Under such conditions, a significant improvement in terms of computational operation is observed.

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