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Analog Computation Using Quantum Structures —— A Promising Computation Architecture for Quantum Processors ——

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SUMMARY Analog computation is a processing method that solves problems utilizing an analogy of a physical system to the problem. As it is based on actual physical effects and not on symbolic operations, it is therefore a promising architecture for quantum processors. This paper presents an idea for relating quantum structures with analog computation. As an instance, a method is proposed for solving an NP-complete (nondeterministic polynomial time complete) problem, the three-color-map problem, by using a quantum-cell circuit. The computing process is parallel and instantaneous, so making it possible to obtain the solution in a short time regardless of the size of the problem.

key words: *analog computation, quantum device, processing architecture, optimization problem*

1. Introduction

One of the goals in quantum electronics is to develop computing systems that can solve given problems by utilizing quantum effects, what I call "quantum processors." To develop such systems, we must first find a computation method or processing one that can be implemented using quantum effects. The purpose of this paper is to introduce a probable candidate for such a processing method. It is *analog computation*. This paper illustrates the concept of analog computation with examples, then presents for future discussion an idea for relating quantum structures with analog computation. I hope that it will stimulate the readers' thinking in this area.

2. What is Analog Computation

Analog computation is a processing method that solves a mathematical problem by applying an analogy of a physical system to the problem. To solve the problem with this method, one prepares an appropriate physical system and represents each problem variable by a physical quantity in the system. If the mathematical relations between the physical quantities are analogous to those of the problem, then he can find the solution to the problem by observing the behavior of the system and measuring the corresponding physical quantities. A processing method based on this principle is called analog computation.

The analog computation is quite different from the commonly used binary-digital computation (Fig. 1). In the digital one, we first devise an algorithm (a set of instructions for finding the solution to a problem), then execute each step of the algorithm in the manner of a Boolean operation under Neumann-type computing architecture. In contrast, an analog computation is concerned with no symbolic Boolean operation; instead it utilizes the properties of a physical system to perform the mathematical operations required for the solution. A unique and important feature of analog computation is that it is based on "live" physical matters and not on symbolic operations; therefore it is probable that quantum effects will be utilized well for implementing analog-computation systems.

In the following, I will describe for explanation two known examples of an analog-computation system. One is a differential analyzer that solves differential equations by using integrators (Sect. 3) and the other is a soap-film system for analyzing the Steiner tree problem (Sect. 4). After that I will present an instance of applying quantum effects to analog computation (Sect. 5). It is the solution to the three-color-map problem that uses quantum-cell circuits.

3. Analog-Computing System for Solving Differential Equations

As a first example of analog computation, I will illus-

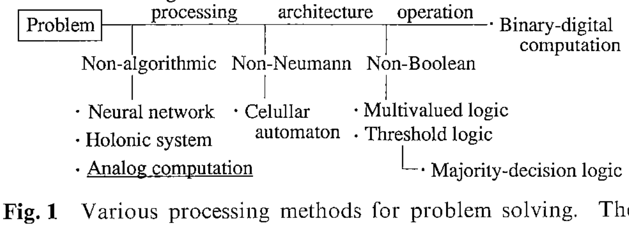

Fig. 1 Various processing methods for problem solving. The commonly used binary-digital computation is based on algorithms, Neumann architecture, and Boolean operation. Analog computation is based on an analogy of a physical system to the problem.

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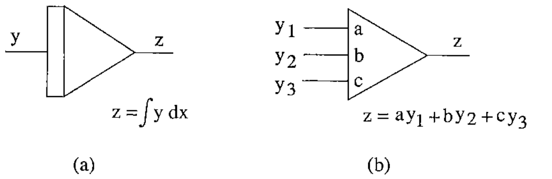

Fig. 2 Basic elements for constructing a system analogous to differential equations. (a) Integrator. (b) Summing amplifier.

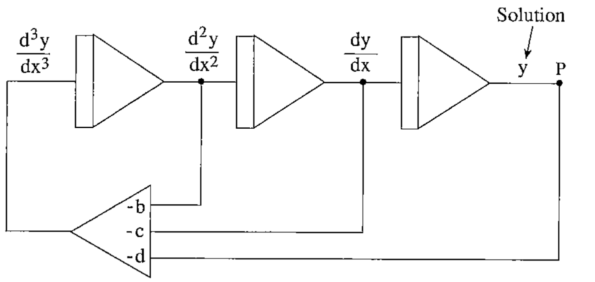

Fig. 3 A physical system setup for the differential equation $d^3y/dx^3+b\ d^2y/dx^2+c\ dy/dx+d\ y=0$. The solution y is obtained on node P.

trate the method for solving differential equations. For example, consider the ordinary differential equation

$$d^3y/dx^3+b\ d^2y/dx^2+c\ dy/dx+d\ y=0.$$

In digital computation, this differential equation is first reduced to simultaneous first-order differential equations, then solved by an application of the numerical techniques such as Runge-Kutta and multistep methods.

The procedure for analog computation differs, as follows. We first construct a physical system analogous to the equation. Such a system requires two basic elements (Fig. 2):

1. An *integrator*, which produces integration of a physical variable with respect to another physical variable;
2. A *summing amplifier*, which produces a weighted sum of input variables.

The differential equation under investigation is implemented by combining three integrators and a summing amplifier to construct a closed-loop system, as illustrated in Fig. 3. We can find the solution to the equation by observing the change of the physical variable on node P. A computing system of this type is called a differential analyzer.

For information, two types of integrators are illustrated in Fig. 4. Both mechanical and electronic differential analyzers were employed in engineering until the 1950's. They could not compete, however, with digital computers because of their fundamental limitation in accuracy and troublesome procedure for system setup (parameter scaling and element connection); therefore they gradually faded into the background. But they can be expected to make a comeback,

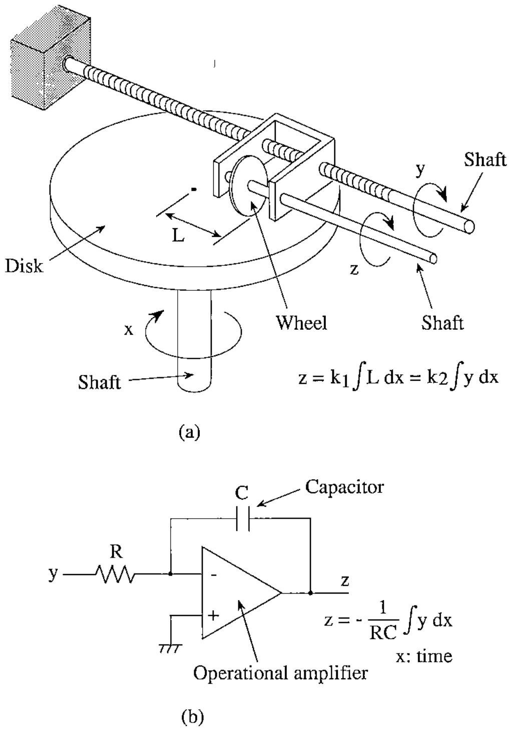

Fig. 4 Two types of integrators. (a) Mechanical wheel-and-disk integrator. The variables x and y are the rotation of shafts. (b) Electronic integrator consisting of an operational amplifier and a capacitor. The independent variable x is time, and the dependent variable y is voltage.

provided that minute and high-speed integrators can be created by use of quantum structures. If a quantum LSI is developed that has a great many integrators on a chip, it will provide an useful tool for quickly solving partial differential equations and integral equations — a task that is difficult for the classical differential analyzers.

4. Soap-Film System for Analyzing the Steiner Tree Problem

Not all problems that can be solved by analog computation are differential equations. Let's turn to another field, the optimization problems.

As an example, consider the following problem (Fig. 5). Connect n points on a plane, using a graph of minimum overall length, with the use of additional junction points allowed. This is called the Steiner tree problem. Plainly expressed, the problem is "to connect n cities by a road network of minimum total length."

This problem is intractable for digital computation. There are many possible graphs with junction points, and we must examine all the possible ones to find the minimum solution. The number of computational steps required increases exponentially with the number n of original points. Indeed, the Steiner tree problem belongs to the class of NP-hard

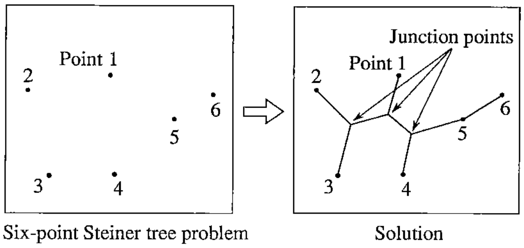

Fig. 5 The Steiner tree problem. Connect given points on a plane with a graph of minimum overall length. This is difficult to solve using existing computers because it requires enormous computing time.

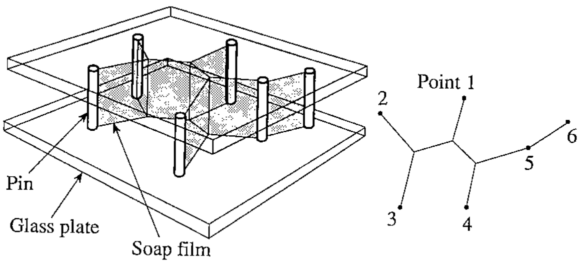

Fig. 6 A soap-film solution to the Steiner tree problem. The problem can be quickly solved by utilizing the equilibrium of a soap-film system. (See Ref. [2].)

(nondeterministic polynomial time hard) problems. Except for inefficient exponential-time procedures, no algorithm is known for the solution. This problem therefore requires enormous computing time to solve and is virtually unsolvable for large values of n. (For details of NP-complete and NP-hard problems, see Ref. [1].)

Nevertheless, there is an ingenious analog-computation method that can quickly solve the problem [2]. We use soap films to make a physical system analogous to the problem (Fig. 6). Prepare two parallel glass plates and insert n pins between the plates to represent the points; then dip the structure into a soap solution and withdraw it. The soap film connects the n pins in the minimum Steiner-tree graph. The computing process is parallel and instantaneous, so we can obtain the solution in very short time regardless of the number n of the pins.

In this analog computation, the energy-minimizing principle is well utilized for problem solving. Any physical system changes its confuguration to decrease its total energy. In liquids at rest, the relevant energy components are the gravitational potential energy and the surface energy. The latter is dominant in a thin soap film, and so a soap-film system changes its configuration to minimize its total area (therefore its length) and thereby its surface energy.

Strictly speaking, it is not possible to be certain, in this system, that the absolute minimum solution can always be obtained. Depending on the angle at which

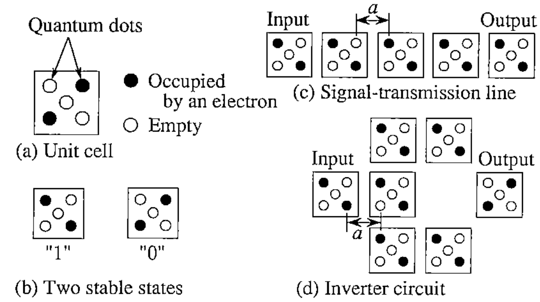

Fig. 7 Quantum-cell circuits. (a) A unit cell consisting of five quantum dots with two electrons. (b) Two polarization states "1" and "0." (b), (c) Two circuit components. The letter a denotes the nearest-neighbor distance. (See Refs. [3] and [4].)

the system is withdrawn from the soap solution, the soap-film network sometimes assumes topologies different from the optimum one that gives the minimum network length. Even in such cases, however, the networks obtained are always nearly equal to the minimum one. Hence it can be said that the system works well in general.

5. Quantum-Cell Circuit for Solving the Three-Color-Map Problem

It is interesting to speculate what analog computations are possible in the quantum-electronics world. Setting aside the issue of the feasibility of constructing actual devices, I present here an instance of applying quantum structures to analog computation. It is the solution to three-color-map problems that utilizes the property of quantum-cell circuits. In the following, I will describe the concept of quantum-cell circuits and the three-color-map problems, and then I will propose a quantum-cell circuit system for solving the problems.

5.1 Quantum-Cell Circuit

The quantum-cell circuit is a logic system composed of quantum cells, each of which consists of coupled quantum dots. This concept was first introduced by Lent and colleagues [3], [4]. They proposed a cell structure that consists of five quantum dots located at the corners and the center of a square (Fig. 7(a)). The cell is occupied by two electrons that are tunneling among the dots within the cell. (A fixed positive charge is also assumed on each dot to maintain charge neutrality in the cell.) Because of Coulomb repulsion, the electrons tend to occupy the diagonal sites in the cell. Therefore, the cell has two stable states of polarization (Fig. 7(b)). By combining the unit cells, various circuit components can be designed. As an example, Fig. 7(c) illustrates a *signal-transmission line* consisting of an array of such cells. The polarized state of the input cell induces the same polarization in all the cells in the line, so that a binary signal is transmitted on the line from one point to another. Figure 7(d) illustrates an *inverter circuit* that produces, in the output, polarity opposite to that of the input. (Other logic circuits have also been designed: see References for details.)

Quantum-cell circuits require several conditions for operation. They work as desired at any time if both of the following two conditions are satisfied:

(a) The interelectron repulsive force between different cells acts only at the distance of the nearest neighbor, denoted by the letter a in the figure, and does not extend further.

(b) The circuit system can be *annealed* into an equilibrium state of global-minimum potential energy†.

The cell circuits can be operated without these conditions, but in that case careful consideration is required as to the arrangement of cells (i.e., to the position of each cell and the distance of cells). For simplicity, the two conditions are assumed in the following.

5.2 Three-Color-Map Problem

Consider the following problem: Can the countries on a given map be colored with *three* colors so that no two countries that share a border have the same color (Fig. 8)? This is called the three-color-map problem and is difficult to solve for a map of many countries. There are colorable maps and uncolorable ones, but we cannot tell whether a given map can be colored before examining all the possible colorings. (The problem is easy if we can use four colors, because it has been proved that four colors suffice for any map.) The three-color-map problem belongs to the class of NP-complete, and is intractable for digital computation because only exponential-time algorithms are known for the solution.

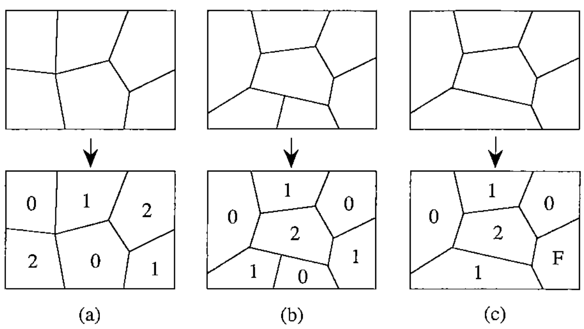

Fig. 8 Coloring of a given map with three colors. (a), (b) Colorable maps. The numbers 0, 1, and 2 represent three colors. (c) An uncolorable map. The trial solution shown fails on reaching region F.

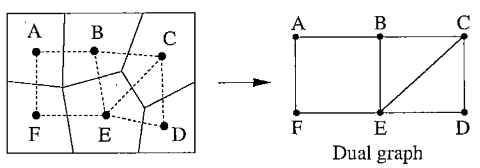

Fig. 9 The dual graph for the map in Fig. 8(a). Each point is colored in one of three colors. Two points connected by a line cannot have the same color.

This problem is reduced to one of graph coloring (Fig. 9). Any map can be converted into a corresponding dual graph by reducing each country to a point and drawing a line between two points if the corresponding countries share a border. Coloring the map is then equivalent to coloring the graph, under the rule that two points connected by a line cannot have the same color.

5.3 Quantum-Cell Circuit System for the Problem Solving

The following describes how to solve the three-color-map problem by using the quantum-cell circuits. Our work is first to construct a quantum-cell circuit analogous to a given map for the problem and then to analyze the problem by using the cell circuit.

5.3.1 Defining Cell Structures

I now introduce two types of quantum cells for constructing the analog system: a *three-dot* and a *two-dot* cell (Fig. 10). The former consists of three quantum dots located at the vertices of an equilateral triangle, and the latter of two quantum dots. Each cell is occupied by an electron that is tunneling among the dots within the cell.

5.3.2 Implementing the Dual Graph by Using the Cells

Taking the map of Fig. 8(a) as an example, we construct the analogous cell circuit for problem solving. The map can be converted into the dual graph of Fig. 9, reducing our task to constructing a cell circuit analogous to the graph.

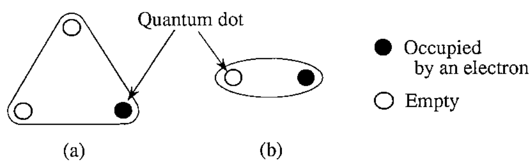

Fig. 10 Quantum cells for the problem-solving system. (a) A three-dot cell with an electron. (b) A two-dot cell with an electron.

† The *annealing of the quantum-cell circuits* has been conceived and studied by Akazawa and colleagues as an indispensable procedure for operating the cell circuits. Their work is under submission.

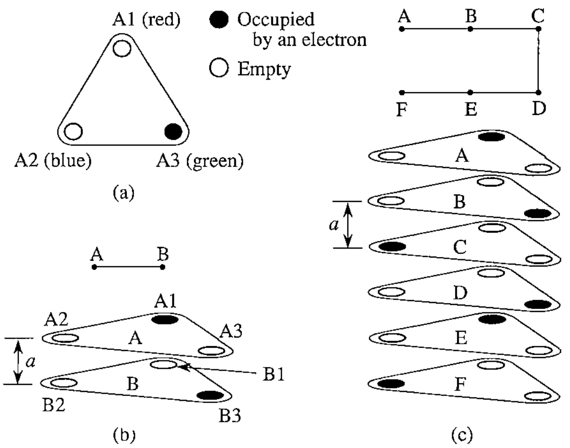

Fig. 11 Construction of a quantum-cell circuit analogous to the three-colorability problem. (a) A three-dot cell representing a point on the graph. (b) A cell circuit analogous to the two connected points A and B. The letter a denotes the nearest-neighbor distance. (c) A cell circuit analogous to the subgraph, consisting of the six points A-F and five lines.

We represent a point on the graph by a three-dot cell (Fig. 11(a)). The three dots, A1–A3, represent three colors; e.g., A1 represents red, A2 blue, and A3 green. The color of the point is equal to the color of the dot that is occupied by an electron; e.g., the point is colored green if an electron is on A3.

We first implement two points A and B connected by a line. This is done by coupling two three-dot cells as illustrated in Fig. 11(b). Interelectron repulsion acts at the nearest-neighbor distance denoted by the letter a in the figure. Therefore two electrons in the coupled two cells cannot occupy the dots that represent the same color; e.g., if the electron in cell A is on dot A1, then the electron in cell B is on a dot of differing color, B2 or B3.

By putting six three-dot cells at regular intervals, we obtain a cell circuit analogous to a subgraph consisting of the six points with five lines (Fig. 11(c)). Electrons in neighboring cells (or coupled cells) cannot occupy dots of the same color —— hence are on dots of differing colors —— because interelectron repulsion acts at the nearest-neighbor distance a.

5. 3. 3 Coupling the Three-Dot Cells to Complete the Cell Circuit

To develop the subgraph circuit of Fig. 11(c) into the complete dual-graph circuit, we must couple the cells A and F, B and E, and C and E. This is done by coupling the corresponding dots, using an array of the two-dot cells. Figure 12 illustrates how to couple two dots (A1, F1) in two three-dot cells (A, F). The coupling array consists of two-dot cells that are lined up at intervals of the nearest-neighbor distance a. Because of interelectron repulsion, any two of the

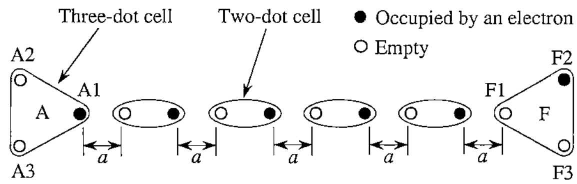

Fig. 12 Coupling two three-dot cells (A and F) with each other, using an array of two-dot cells. A coupling structure for dots A1 and F1 is illustrated. The letter a denotes the nearest-neighbor distance. Coupling for two other dot pairs, A2–F2 and A3–F3, are also needed.

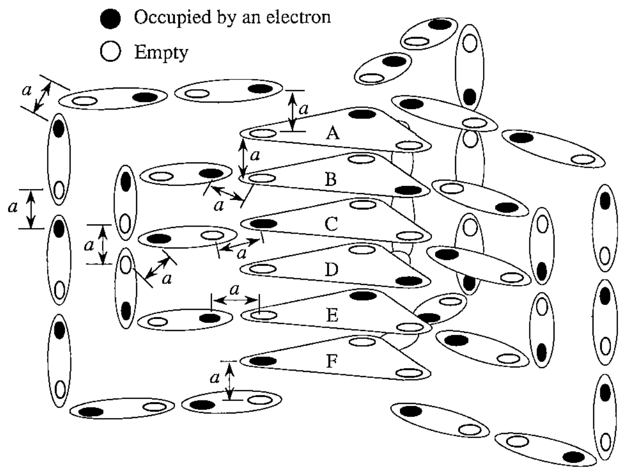

Fig. 13 The analog-computation system for solving the three-color-map problem for the graph in Fig. 9 (or the map of Fig. 8 (a)). The letter a denotes the nearest-neighbor distance.

electrons cannot face each other at the nearest-neighbor distance; therefore if the electron in the cell A occupies dot A1, then the electron in the cell F cannot occupy dot F1. Coupling for two other dot pairs, A2–F2 and A3–F3, are also needed to couple the cells A and F.

The completed cell circuit is illustrated in Fig. 13. Using the same procedure, we can construct an analogous cell circuit for any problem map.

5. 3. 4 Solving the problem by using the cell circuit

The three-color-map problem asks whether the given map is colorable, and the answer is either "*yes*" or "*no*". To solve the problem by using the completed cell circuit, we first anneal the circuit into an equilibrium state with global-minimum energy, then check to see whether electrons in any coupled three-dot cells are on dots of *differing* colors. If they are, the answer is "*yes*" and colors of the occupied dots indicate the coloring in which the map can be colored. If they are not, the answer is "*no*".

This solution is based on the following two properties:

(a) "A map is colorable" is equivalent to "the analogous cell circuit has one or more arrangements of electrons in which no two electrons face each other at the nearest neighbor distance." We call the state of such arrangements the "*ground state*."

(b) There is repulsive-force energy between two electrons that face each other at the nearest-neighbor distance. Therefore a cell circuit will change its state to minimize the number of such electron pairs.

In the process of annealing, a cell circuit for a colorable map settles down to the ground state, and we will find that electrons in any coupled cells are on dots of *differing* colors. In a circuit for an uncolorable map, on the other hand, the ground state cannot be attained, and we will find that electrons in one or more coupled-cell pairs are on the dots of the *same* color.

A similar solution using quantum-cell circuits should exist for other NP-complete problems because every NP-complete problem belongs to the same class and one can be converted into another.

6. Conclusion

Analog computation is a processing method that solves mathematical problems by applying an analogy of a physical system to the problem. An idea for relating quantum structures with analog computation is presented. As an instance, a solution to the three-color-map problem by using a quantum-cell circuit is proposed. The computing process is parallel and instantaneous, making it possible to obtain the solution quickly. Analog computation is a promising architecture for quantum processors.

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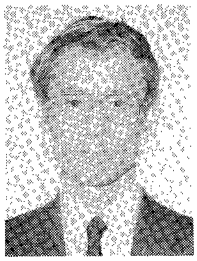

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